NOTES ON LANDING AND TAKEOFF ISSUES FOR A TRI-SERVICE STRIKE FIGHTER

by
William D. O'Neil
Center for Naval Analyses

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These notes explore a specific subset of the issues involved in defining a tri-service strike fighter. Obviously, one set of issues concerns the requirements for range and combat performance, which the services have had a notably difficult time agreeing on in the past. But this is not the set of issues I propose to take up here. Instead, I am going to address the seemingly more mundane question of takeoff and landing — launch and recovery, in naval terms. These are strictly personal notes, which do not represent finished research and do not present a CNA position.

Varying Requirements. The Navy, of course, has a hard and fast requirement that its tactical aircraft be suitable for operations from ship. Since it has a substantial stock of aircraft carriers and since it would be hugely expensive to replace them, or even to make major modifications to them, it makes sense to require that Navy aircraft be operable from existing carriers, with at most minor alterations to the ships.

In addition to operating standard Navy types from expeditionary airfields, the Marine Corps operates AV-8B V/STOL (vertical and/or short takeoff and landing) light attack aircraft from improvised short and very short strips close to the front lines, giving better responsiveness than it possible from an airfield which may have to be hundreds of miles to the rear. Also, the AV-8B can operate from LPH/LHA/LHD class ships. In practice, the AV-8B is almost always operated in a STOVL (short takeoff with vertical landing) mode, since its payload with vertical takeoff is too low to meet real operational needs. As the basic AV-8B design dates back more than 30 years and as the existing Marine fleet of this aircraft is aging, the Marines are looking for a replacement with comparable or better takeoff and landing performance.

Air Force requirements are for aircraft which can operate from civil or military airfields. Almost all commercial airfields have at least one hard-surfaced runway over 4,000 feet in length (which may, however, be gravel-surfaced in Arctic regions), and most nowadays have at least an 8,000 foot strip. But the Air Force must reckon on the possibility of having to operate from bases under attack by TacAir (tactical air forces) or missiles, which may damage runways. For this reason, an ability to operate from a short undamaged section of a longer runway is desirable. Also, the aircraft may need to land and take off from strips which have been repaired rather roughly, putting extra stress on landing gear and aircraft structure.

So the Marines feel they need a STOVL aircraft to replace the AV-8B. The Air Force sees a clear requirement for a CTOL (conventional takeoff and landing) aircraft to replace the F-16. And the Navy feels it must have an aircraft which is compatible with catapult launch and arrested recovery (CLAR) to complement the F/A-18E/F.

The Costs of Commonality. At first glance, this seems to leave the Marines as the odd service: in principle a CLAR aircraft for the Navy can meet Air Force CTOL requirements also, just as the F-4 and A-7 CLAR aircraft served the Air Force. In practice, this notion encounters a good deal of resistance from the Air Force. The problem is that CLAR operation puts stresses on the aircraft's structure far in excess of those for CTOL operation. To accommodate these stresses, the structure must be made stronger, resulting in a weight penalty. It is difficult to pin down the exact value of this penalty, but experienced aircraft designers believe that it is generally of the order of 10% of the empty weight.

Is this serious? It certainly can be. Imagine that the empty weight is half of the loaded takeoff weight for a strike mission (a typical value). Then the CLAR weight increment amounts to subtracting 5% of takeoff weight from the aircraft's useful load. Suppose we want to get this amount of useful load back without sacrificing range or combat performance, assuming that we are at a stage of the design where we can enlarge the aircraft at will. For any aircraft design there will be a "weight multiple," a factor representing how many pounds must be added to total weight in order to add a pound without sacrificing performance. This accounts for the extra structure to carry the added weight, the extra engine thrust required to push the added weight and structure, and the additional fuel required to carry all this. The weight multiple varies with the design and the location of the weight on the aircraft, but 4:1 is a fairly typical value for aircraft of this class. If that is the case, then we will need to add $4 \times 5\% = 20\%$ to the gross weight of our hypothetical strike fighter in order to recover what was lost in adding CLAR capabilities! All else being equal, this will add 20% or so to the acquisition cost, and somewhat less to the O&S (operations and support) costs. In all, we are probably talking something on the order of a 15% increment in LCC (life-cycle costs), or about 15% fewer aircraft for the same budget. The Air Force has not been inclined to pay such penalties for the doubtful benefit of sharing aircraft with the Navy, and this has been one of the causes for the lack of successful joint USAF/USN aircraft projects. (Of course there is a savings to the Air Force in acquisition costs per unit from producing in larger quantities, but in most cases this will not amount to as much as 15% of LCC.)

Attempting to meet all three services' needs by building a common STOVL aircraft could involve still more daunting problems. The weight penalties of STOVL operation over a CTOL aircraft of otherwise comparable performance can be very large — as much as 100% in some cases. This is why the Air Force has never shown much interest in STOVL or other variants of V/STOL.

All this at first sight seems to argue for saying "to hell with it" and allowing each service to build an aircraft to suit its needs. Under current conditions, however, it may be neither economically nor politically feasible to build more than one type. This problem might be resolved by a death battle between the services to determine which shall be the one to continue to operate TacAir. But this prospect seems unattractive enough to make it worthwhile to devote some further thought to the problem.

The Savings from Commonality. The savings from commonality are much taken for granted, sometimes discussed, and very rarely quantified. Annex A is an attempt to consider them quantitatively, if very crudely so. It presents a highly simplified model of how costs might vary with procurement quantity as well as with the number of different service models within one overall program. Fundamentally, it assumes that as production volume is doubled, the procurement cost increases at some fixed fraction of 2.0. That is, if

$P(n)$	is the procurement cost for the first n units
p_1	is the procurement cost for the initial unit = $P(1)$
r	is the fixed rate at which procurement costs fall, on a cumulative average basis, with doubling of quantity

then we assume that

$$P(n) = p_1 \times n^{1+\frac{\ln r}{\ln 2}}$$

It will be seen that

$$P(2n) = 2 \times r \times p_1 \times n^{1+\frac{\ln r}{\ln 2}} = 2 \times r \times P(n)$$

thus demonstrating that this formulation does meet our assumption that as production volume is doubled, the procurement cost increases at a fixed fraction, r, of 2.0. This is roughly in accord with experience, with r often referred to as the *learning rate* (although technically the learning rate refers to assembly man-hour reductions, not those of cost). Values of r typically have been in the range of 80% to 85% for aircraft, but there is some speculation that r is tending to rise (*i.e.*, get less favorable to reduced costs) over time.

Additionally, Tab A assumes that as service sub-types are added to a combined aircraft program, the RDT&E costs increase with some power of the number of sub-types — the power is taken as 0.6 in the Tab. The basic RDT&E cost (with just one type) is taken as 40 times the first unit production cost, p_1. All this reflects the fact that aircraft RDT&E costs are largely spent on building prototypes. If additional sub-types having a high degree of commonality are added to a program, it is by no means necessary to double the number of prototypes, thus bringing a considerable savings. Note that I have assumed that the production cost of the first unit is p_1 = $360 million. While this seems extravagant, it is in fact a very modest value compared to the $476 million I estimate for the F-22.

As will be seen from Figure A-1, the savings from a combined program could reasonably be expected to lie in the range of $30 billion to $40 billion, making such an approach seem very attractive indeed. Examination of Figures A-2 and A-4, however, shows that if values of r creep up, and particularly if a combined program suffers from higher values of r than separate programs, then the savings might dwindle substantially, or even vanish.

(Tab A also contains an example to show the very powerful temptations services — particularly the Air Force — may feel to avoid participating in a combined program. Under the assumptions of this example (which are not too unrealistic) the Air Force might believe it stood to save some $6.4 billion by buying more F-22s rather than contributing to a combined program. Of course this would raise the costs to the remaining services — by more than $14 billion in this example — and severely erode their incentives to pursue a common program.)

The very substantial savings which might be possible from a combined program clearly justify, in my view, an attempt to formulate such a program. But the fragility of the savings counsels great care.

The Physics of Takeoff and Landing (Greatly Simplified). Let us begin by considering the physics of takeoff and landing in very simplified form. It can be shown that the lift of an aircraft can be expressed as

$$\mathit{Lift} = \frac{\mathit{Air\ density}}{2} \times \mathit{Lift\ coefficient} \times \mathit{Wing\ area} \times \mathit{Speed}^2$$

Here the *lift coefficient* is a non-dimensional quantity which expresses the aircraft's effectiveness at converting forward motion into lift and it is assumed that some consistent set of units is used for the other quantities.

In uniform near-level flight, we must have

$$\mathit{Lift} \approx \mathit{Weight}$$

When the two are equal, we can write

$$\frac{\mathit{Air\ density}}{2} \times \frac{\mathit{Lift\ coefficient}}{\mathit{Wing\ loading}} \times \mathit{Speed}^2 = 1$$

where the *wing loading* is the weight per unit area of wing. Obviously, this implies that if the aircraft varies its speed at constant air density, the quantity *lift coefficient / wing loading* must vary inversely as the square of speed. In practice, it is largely the lift coefficient which is varied.

At sea level we take the density of air to be its standard value of 0.0023769 $slugs/ft^3$. If we have an aircraft with a wing loading of 80 lb/ft^2 and we want it to fly at a sea level speed of 1,060 ft/s (Mach 0.95) then we find

$$\begin{aligned} \textit{Lift coefficient} &= \frac{\textit{Wing loading}}{\textit{Speed}^2 \times \frac{\textit{Air density}}{2}} \\ &= \frac{80}{\textit{Speed}^2 \times \frac{0.0023769}{2}} \\ &= 67{,}315 \times \textit{Speed}^{-2} \\ &= 0.06 \end{aligned}$$

where *speed* is expressed in ft/s.

Generating Lift. If we want our aircraft to stall at a speed of 169 ft/s = 100 knots, then we find

$$\textit{Lift coefficient} = 2.36$$

In addition, we will want to design our aircraft to be able to generate negative lift coefficients for maneuvering. In all, a typical tactical aircraft will operate at a minimum lift coefficient in the -0.8 to -1.0 range and at a maximum of 1.5 to 2.5.

Any flight vehicle generates its lift by imparting a downward component of velocity on the air about it. With an airplane, this is principally the function of the wings, which deflect the air somewhat downward as they pass through it. The main way that an airplane increases its lift coefficient is to increase its angle of attack (the angle between the plane of the wing and the aircraft's velocity vector with respect to the air). This increases the downward deflection of the air and thus increases lift — up to a point. For as the angle of attack increases beyond a certain angle (varying from one airfoil to another, and with a variety of circumstances) the smooth and steady flow of the air over the wing breaks down and the air flow separates from the upper wing surface. As angle of attack increases further the area of separation covers a larger portion of the wing surface and degrades the wing's ability to deflect air downward. Beyond a certain angle, further increases bring not an increase but a decrease in lift, and the wing is said to be stalled.

The kinds of wings which are suitable for tactical aircraft are rarely able to generate lift coefficients greater than 1.0 simply by increasing angle of attack. When greater lift coefficients are required, three basic methods are most often used (often in combination):

- Increase the camber (overall curvature) of the wing profile by downward deflection of a flap at its trailing edge.

- Increase the angle of attack at which flow separates by leading-edge flaps or slots, turbulence generators, or other means to increase the energy of the flow.

- Increase the effective lifting surface area by extending flaps forward and/or aft of the wing.

It is possible by these means to generate lift coefficients as great as 4.0 — but not on practical tactical aircraft[1,2]. In order to achieve transonic or supersonic flight, a tactical aircraft must have a thin wing (maximum thickness typically no more than 5% of chord for relatively straight wing segments and no more than 10% even for the most highly swept) with either high sweep or a sharp leading edge. This both reduces the effectiveness of high-lift devices and also leaves minimal room to fit them. In addition, these aircraft can not afford the drag penalties associated with the external flap tracks which are needed for the most effective kinds of devices. As mentioned before, few tactical aircraft can achieve lift coefficients in excess of 2.5, and most can not better 2.0. We have seen above how this leads to stalling speeds not much under 100 knots, with corresponding approach speeds of 120 knots or more.

Some aircraft achieve lift coefficients of 10.0 or more by other means. Usually, these involve blowing jet exhaust or high-pressure bleed air from slots on the top of the wing, or immediately in front of the flaps. This requires a system of ducts which seems difficult to fit within a thin wing. Some tactical aircraft (e.g, the F-4 and A-6) have made use of blowing for boundary layer control, but this involves relatively small quantities of bleed air.

As will be seen presently, there would be much value in being able to achieve maximum lift coefficients of from 2.75 to 4.0 in tactical aircraft. Is this feasible? High lift devices can, as described above, produce lift coefficients of 10.0 or so, but they work best on high aspect ratio, relatively thick wings: tactical aircraft need low aspect ratios with thin wings. One approach is to use variable sweep to permit low aspect ratio for cruise and high aspect ratio for landing and takeoff, as in the F-14. This also has the effect of increasing the streamwise thickness/chord ratio as the wing is unswept. It is likely that a variable sweep wing with mechanical lift devices could be made to provide maximum lift coefficients as great as 2.75 without too much compromise or penalty.

As mentioned, the most powerful means of achieving high lift involve blowing large volumes of gas through or over flap systems. In transport type aircraft this is done with wing-mounted engines — not a feasible proposition for transonic tactical aircraft. The alternative is ducting the gas from the engine efflux out along the wing. While the thin wing of a tactical aircraft can not accommodate a duct in its cruise condition, it may be possible to use extensible ducts at low speeds — a variable cross-section wing. The Jacobs-Hurkamp jet flap, with its variable-area duct, represents one possible approach[3]. Given the very large volumes of exhaust gas available in a high-performance tactical aircraft, it seems likely that such schemes could be made to yield lift coefficients as high as 4.0 even on highly swept low aspect ratio wings, although it is not so clear what the penalties might be.

Accelerations and Loads. A CTOL or CLAR aircraft has to accelerate from rest to flying speed while on the ground for takeoff and reverse the procedure for landing. If we imagine that only a certain distance is available for this purpose and assume that the aircraft accelerates (or decelerates) uniformly over this distance, we find

$$Acceleration_{required} = \frac{Flying\ speed^2}{2 \times Distance}$$

Obviously, since

$$Force = Mass \times Acceleration$$

the load put on the aircraft's structure is directly related to the *acceleration* and, hence, to the square of the *flying speed*. These loads can be pretty significant for CLAR operation. For instance, in launching an aircraft to an end speed of 135 knots = 228 ft/s in a distance of 304 ft (which is the stroke of the current C13-1 catapult[4]), the catapult must accelerate it at an average of 85.5 ft/sec^2 = 2.65 *g*. In practice the actual average catapulting loads on the aircraft are somewhat less, since some acceleration is also provided by engine thrust. On the other hand, the actual peak loads are typically considerably higher, as the catapult is not able to apply a constant acceleration over its full stroke. Catapult loads are greatest for lower-powered attack aircraft launching at full load, when their thrust/weight ratio is likely to be no greater than 0.3 or so. Longitudinal loads for arrested recovery are not quite so high, since the speeds in general are a bit lower and the distance a bit longer, but are still very significant structurally.

In addition to the longitudinal loads, arrested landings involve substantial vertical loads as the wheels strike the deck. To be recovered rapidly and safely, aircraft making an arrested landing must be flown right down to the deck, with no flare, and this is the U.S. Navy's standard practice. This can mean a pretty high vertical velocity at touchdown. For instance, if the aircraft is flying a standard 3½° glideslope at a speed of 120 knots = 203 ft/s then its vertical velocity at touchdown will be

$$Speed_{vertical} = 203 ft/s \times \sin 3.5° = 12.4 ft/s$$

If the stroke of the landing gear under this load is 1½ feet, then the average acceleration will be

$$Acceleration_{vertical} = \frac{Speed^2_{vertical}}{2 \times stroke} = 51.3 ft/s = 1.6g$$

This is considerably greater than the normal maximum vertical load in a CTOL landing, despite the long stroke assumed for the gear, and requires additional structure.

Lightening the Load. These loads can be ameliorated by reducing the speed at which the aircraft lands and takes off. Let us envision a tactical aircraft with a takeoff wing loading of 100 lb/ft^2 and a landing wing loading of 65 lb/ft^2 (both somewhat lower than typical for the F/A-18), and a takeoff thrust/weight of 0.75 (somewhat better than typical for the F/A-18 in strike loadings). And let us imagine that the maximum lift coefficient it can generate is 1.75 (comparable to the F/A-18). We know that

$$Speed = \sqrt{\frac{Wing\,loading}{Lift\,coefficient \times \frac{Air\,density}{2}}}$$

Using this, we get stalling speeds of 219 ft/s = 130 knots for launch and 177 ft/s = 120 knots for recovery. We will assume that the actual flying speed at the end of the catapult stroke is 20% greater than the stalling speed in order to allow adequate margins, and similarly for touchdown speed margin.

If we assume 15 knots = 25 ft/s wind over deck for launch and recovery, a catapult stroke of 304 feet on launch, and an arresting gear runout of 320 feet, then we find that the average catapult acceleration is

$$\begin{aligned} Acceleration_{catapult} &= \frac{(Launch\,speed - Wind\,over\,deck)^2}{2 \times Catapult\,stroke} - \frac{Thrust}{Weight}g \\ &= 68.8 ft/s^2 = 2.14g \end{aligned}$$

And the average arresting deceleration is

$$\begin{aligned} Deceleration_{arresting} &= \frac{(Recovery\,speed - Wind\,over\,deck)^2}{2 \times Runout} \\ &= 54.7 ft/s^2 = 1.70g \end{aligned}$$

In fact, these formulas neglect many important second order effects and must not be taken as anything like exact — and as indicated before, they represent only average accelerations at best. They are sufficient to establish overall trends, however.

Calculating Takeoff and Landing Runs. It will be useful to be able to calculate runs required for normal takeoff and landing from a runway. We will first assume no wind and sea level operation. The takeoff roll is the distance required for the aircraft to accelerate from rest to flying speed, plus the further distance it remains on the ground while rotating to takeoff angle of attack. (The aircraft normally accelerates at an angle which produces little lift in order to minimize drag due to lift. In order to get airborne, the nose is then raised to increase the lift coefficient to near its maximum.) The discussion here roughly follows that of a standard text on the subject of aircraft design[5].

The accelerating force is of course the net of thrust over drag. The thrust includes the engine thrust as well (in the case of a catapult launch) as the catapult thrust. Drag includes all the various aerodynamic drag components associated with flight, plus the drag increments of the flaps (which are usually deflected for takeoff in order to increase lift) and landing gear, plus the rolling resistance of the landing gear. In general, all of these terms will vary with speed during the course of the takeoff run, making it necessary to perform an integration to obtain the ground run.

However, as we do not know any of the terms with much accuracy and do not need a precise value in any event, we will fall back on a rough approximation often used in preliminary calculations:

$$Ground\,run = \frac{Takeoff\,speed^2}{2 \times Ground\,acceleration}$$

where *ground acceleration* is taken as constant. When *ground acceleration* is taken as the geometric mean of the acceleration over the ground run, the error is generally not too great.

Obviously,

$$Ground\,acceleration = \frac{Ground\,thrust - Rolling\,drag}{Weight} \times g$$

Following usual conventions, we will write

$$Rolling\,drag = \frac{Air\,density}{2} \times Speed^2 \times Wing\,area \times Drag\,coefficient + Wheel\,drag$$

I will use the F-4C as an example to build the method around, because I happen to have fairly good data on it. At a typical attack mission weight of 59,453 lb, with a wide assortment of external stores, the F-4C has a power-off stalling speed of 154.5 knots = 260.8 ft/s in takeoff configuration[6]. If we assume (as is customary) that the takeoff speed is 20% greater than the stall speed, we find *takeoff speed* = 185.4 knots = 312.9 ft/s. We will take the mean speed for calculations to be 70% of this, or *mean speed* = 129.8 knots = 219.0 ft/s.

For the F-4C, we have[7]

$$Drag\,coefficient = 0.024 + 0.169 \times Lift\,coefficient^2$$

in normal subsonic flight at Mach 0.8. Normally, the zero-lift drag coefficient would fall off somewhat at slower speeds, but we will take this as representing the aerodynamic drag coefficient during takeoff in an effort to account for the added drag of extended flaps and landing gear. While the angle of attack is held to zero or even slightly negative, the flaps will be extended and the camber they induce will generate lift even at negative angle of attack, so the second term is significant. (In fact, the camber induced by the flaps will change the form of the equation somewhat, but this is a refinement we will ignore in this discussion.)

The wheel drag is typically taken as some constant (depending on the kind of surface) times the weight on the wheels. Of course the weight on the wheels is the gross weight less lift, so

$$\textit{Wheel drag} = \textit{Wheel drag factor} \times \left(\textit{Gross weight} - \frac{\textit{Airdensity}}{2} \times \textit{Lift coefficient} \times \textit{Wing area} \times \textit{Speed}^2\right)$$

For concrete or similar surfaces, we can take *wheel drag factor* = 0.04. In total, we get for the F-4C

$$\begin{aligned}\textit{Rolling drag} = 0.04 \times \textit{Gross weight} + \frac{\textit{Airdensity}}{2} \times \textit{Wing area} \times \textit{Speed}^2 \times \\ (0.024 - 0.04 \times \textit{Lift coefficient} + 0.169 \times \textit{Lift coefficient}^2)\end{aligned}$$

To find the lift coefficient which minimizes rolling drag, we will differentiate with respect to lift coefficient and set the derivative to zero. Abbreviating D_R for *rolling drag* and C_L for *lift coefficient*, we find

$$\frac{dD_R}{dC_L} = \frac{\textit{Airdensity}}{2} \times \textit{Wing area} \times \textit{Speed}^2 \times (-0.04 + 0.169\, C_L) = 0$$

so

$$\textit{Lift coefficient} = \frac{0.04}{0.169} = 0.24$$

is optimum for takeoff in this aircraft under the conditions assumed. This can be achieved by partial extension of flaps. With this lift coefficient, we find *rolling drag* = 3,110 lb, which is 5% of gross weight. In the absence of better data, we will take 5% of gross weight as typical of rolling drag for all aircraft.

The uninstalled static thrust of the F-4C at maximum power (i.e., with full afterburning) is 34,000 lb[8]. If we take 95% of this as the installed thrust we get 32,300 lb = 0.54 × *weight*, and we will assume that this is constant throughout the takeoff run. Thus we find

$$\begin{aligned}\textit{Ground acceleration} &= (0.54 - 0.05)g \\ &= 0.49g = 15.8\,ft/s^2\end{aligned}$$

and

$$\textit{Ground run} = 3{,}090\,ft$$

To get takeoff run, we must add the distance covered by the aircraft during the rotation, usually taken to be 3 seconds at takeoff speed. Thus we find

$$Takeoff\ run = 3{,}090ft + 940ft = 4{,}030ft$$

The true figure is 4,260 ft, or 6% greater[9]. Table 1 compares calculated values from this model for various aircraft with actual takeoff runs as reported in Standard Aircraft Characteristics (SAC) charts. As will be seen, there is reasonable agreement for most aircraft, with the calculated values being somewhat conservative overall. Thus this very simplified model is appropriate for analyzing overall trends.

Landing Run. We will use a very similar methodology for the simpler problem of estimating landing run. It is normally assumed that the aircraft rolls freely for a certain interval after touchdown. This is usually taken to be two or three seconds, and we will assume two for the pragmatic reason that it will lead to better correlation between our model and actual data, as we shall see. During this roll, the aircraft decelerates slightly, under the influence of

$$Rolling\ deceleration = \frac{Rolling\ drag}{Weight} \times g$$

and travels a distance equal to

$$Free\ roll = Landing\ speed \times \Delta t - \tfrac{1}{2} Rolling\ deceleration \times \Delta t^2$$

where Δt is the free roll duration of 2 seconds. At the end of this period, speed will have fallen off slightly due to the rolling deceleration.

Following the free roll, brakes are applied. It is usual to take the braking coefficient as that for wet concrete, about 0.3. Thus deceleration increases sharply at this point to

$$Braking\ deceleration = \left(Braking\ coefficient + \frac{Rolling\ drag}{Weight}\right) g$$

Then

$$Braked\ roll = \frac{(Landing\ speed - \Delta t \times Rolling\ deceleration)^2}{2 \times Braking\ deceleration}$$

and

$$Landing\ roll = Braked\ roll + Free\ roll$$

As Table 1 shows, this produces very good agreement with actual values from SAC charts. The agreement is appreciably better than for takeoff, no doubt because the forces during the landing phase do not vary so much.

Thrust Reversing. Thrust reversing has reportedly been applied in the Swedish *Viggen* combat aircraft where takeoff and landing performance is considered very important — the reversers are automatically engaged immediately on nose-gear touchdown, leading to a landing run of approximately 1,600 feet[10]. (Calculations of landing and takeoff performance for the *Viggen* are included in Table 1, based on rough data. The landing distance calculation suggests that the thrust reverser cuts landing distance by roughly 30%.) Thrust reversing was seriously considered for the USAF F-22, but was not ultimately adopted (principally because the design had difficulty accepting extra weight so far aft.) On transport aircraft, thrust reversers can supply up to 40% of the engine static thrust, but this is typically no more than a reversing thrust/weight of 0.15 or so, owing to the low installed thrust relative to the weight of these heavy aircraft.

If we assume that braking and thrust reversal cut in at the same time, we can easily recast our equations by taking

$$\textit{Reversing deceleration} = \left(\textit{Braking coefficient} + \textit{Reversing coefficient} + \frac{\textit{Rolling drag}}{\textit{Weight}}\right) g$$

Variation in Takeoff and Landing Performance with Lift Coefficient. In the accompanying Table 2, these methods are applied for a family of aircraft with equal wing loading and thrust/weight but differing maximum lift coefficient. (For takeoff and landing runs two figures are shown: the deck run in which 15 knots of wind over deck is assumed, and a land run in which no wind is assumed.) A thrust reversing case is included in which it is assumed that the reversing thrust/weight if 0.15. For a high powered tactical aircraft with installed thrust typically greater than usual landing weights this allows for a reasonably light and uncomplicated reverser. It produces reductions in landing roll of roughly 30%, as with the *Viggen*. Of course these highly simplified and idealized calculations neglect many secondary but significant factors. Despite their limitations, however, the figures provide a useful indication of trends:

- Looking first at the column representing catapult acceleration, we see that for lift coefficients over 5.5, the acceleration goes to zero. This means that the high lift aircraft are capable of lifting off under their own power in less than 304 feet, with no catapult assistance.

- For lift coefficients over 7.25, the arresting gear applies no more deceleration than could be achieved with braking alone (i.e., 0.3g).

- Even a moderate increase in lift coefficient reduces catapult acceleration and arresting deceleration significantly. For instance, increasing maximum lift coefficient from 1.75 to 2.75 cuts catapult acceleration by 54% and arresting deceleration by 41%.

- For zero wind, braked landing runs are about 62% greater than takeoff runs for this high thrust/weight aircraft. But if thrust reversers of moderate effectiveness are employed then the landing run is very close to the takeoff run.

Ski Jump Launch. There is another option for launch from a deck (or on land): a ski jump. In a ski jump takeoff, the aircraft begins its takeoff run on the level and accelerates to a speed well below flying speed before it starts up the ramp of the ski jump. In ascending the ramp (typically set for an exit angle of 6° to 12° above horizontal), some of the aircraft's forward velocity gets converted into vertical velocity or initial rate of climb. The optimal airspeed at the end of the ramp depends on aircraft characteristics and ramp angle, but for high performance aircraft it can be as little as half the normal takeoff speed. Thus as the aircraft leaves the ramp it lacks full flying speed and tends to sink. But it takes some time for this downward acceleration to use up the initial rate of climb, and during this time the aircraft continues to accelerate forward. By the time the rate of climb has fallen to zero the aircraft is considerably higher than the end of the ramp and has reached full flying speed.

Ski jump takeoffs are usually associated with the AV-8B Harrier. The AV-8B derives additional benefit in ski jump takeoff from its ability to deflect thrust downward, and particularly needs the performance benefits that ski jump launch brings, but the principles are no different from those involved with any other high-performance aircraft (despite what has been said by some authorities). Several foreign navies have fitted ski jumps to the bows of Harrier-carrying ships, with good results.

As is well known, the now-defunct Soviet Navy built several aircraft carriers of basically conventional design, but substituting ski jump ramps for catapults. The Soviets apparently intended to operate high-performance interceptors from these ships, developing a version of the Su-27 FLANKER for the purpose. The fate of these ships is uncertain, but I am not aware of any problems with the ski jump launch.

The U.S. Navy built a land-based test ramp at NATC Patuxent River and used it to test several high-performance aircraft with ramp exit angles of up to 9°[11]. These tests were quite successful and revealed no significant problems. (Publication of this research may have influenced the Soviet decision to use ski jumps on their ships.) Under conditions roughly equivalent to those envisioned in Table 2, these tests showed that reductions in takeoff run of 60% or more could be achieved. As can be seen from Table 2, this means that high-performance aircraft could take off with a ski jump in little or no more distance than is needed for catapult launch.

Some lower-thrust current carrier-based aircraft, however, might not be capable of ski-jump launch within the few hundred feet of deck roll which can be achieved on a carrier. For this reason, if ski-jumps were to introduced, it might have to be done in a way that would allow normal catapult launches to proceed at the same time. RADM George Jessen, USN (Ret.) has suggested that this might be done by installing a ski-jump ramp in place of the catapults at the forward end of the angle deck, leaving in place the bow catapults[12]. This seems like a plausible scheme (since the ramp ought to have favorable effects on bolter performance), but it has not been worked out in detail or tested.

Effects on Carrier Sortie Rates. Analyses conducted by investigators at NAEC Lakehurst compared the effects on sortie rates of various launch and recovery schemes[13,14]. They concluded that a V/STOL airwing would be able to generate significantly more strike sorties in a day than would an airwing of otherwise comparable CLAR aircraft. This was because the much smaller landing area for a V/STOL aircraft made it possible to recover aircraft far more rapidly. The advantage declined for longer strike radii, reflecting the decreasing frequency of landing operations for long-range operations. Further work by CNA analysts has greatly extended but generally confirmed the NAEC conclusions[15,16]. As yet, these analytical predictions have not been tested in actual operation.

RADM Jessen has speculated that operations involving ski-jump launch from a ramp on the angled deck might also lend themselves to higher sortie rates by allowing a land-based "quick-turn" style of operation without the need for respotting the deck. These ideas have not been analyzed in detail, but if they worked out it seems that they should result in a sortie rate intermediate between that for CLAR operation and that for STOVL operation.

The Case of the *Rafale*. A contemporary effort to develop a common family of tactical aircraft is of interest[17]. The French government has been developing a new supersonic light strike fighter, the Dassault *Rafale* ("squall" or "burst" (as of firing)), which, in different versions, is to serve both in a land-based role with the *Armée de l'Air* (French Air Force) and from carriers with the *Aéronavale* (French Naval Air Service). While the land and sea versions are not identical, it is officially claimed that commonality is 80% in terms of empty weight and 90% in terms of cost. I have not seen any indications that this program is in danger of being cancelled, even though its costs must seem burdensome and its export prospects rather questionable in the present defense environment.

The configuration adopted for the *Rafale* is one common to a number of recent light strike fighter projects, including the multinational European Fighter Aircraft (EFA), the Swedish Saab *Gripen,* and the uncompleted Israeli IAI *Lavi*; I believe it was introduced with the Saab *Viggen,* which first flew in 1967. This involves a mainplane with a delta planform (normally a somewhat modified delta) and a foreplane (or canard) mounted just ahead of and somewhat above the mainplane. The merits of this delta-canard configuration (like everything else in fighter design) are hotly debated, and I have known very capable and experienced designers on both sides. But there is no denying its popularity, at least in Europe. My own evaluation is that it is a very attractive configuration for a light strike fighter, but less so for larger aircraft.

As with any delta, this configuration tends to optimize for a rather low wing loading. For the *Rafale,* typical takeoff wing loadings for a strike mission appear to be about 90 lb/ft^2, with about 50 lb/ft^2 for landing. In fact, the real loading on the wing in landing trim is even lower than this reveals, since the foreplane is flown at quite a high angle of attack for the approach and probably contributes something like 10% of the total lift. The wing has leading-edge slats (which are probably of no more than modest effectiveness, given the leading-edge sweep of about 45°) but no trailing-edge flaps (like most deltas, its entire trailing edge is taken by elevons which are needed both for pitch and roll control authority and can not contribute much to increasing camber for high lift). Thus the maximum lift coefficient is apparently limited to about 1.7.

Even this modest value, however, is enough on so lightly-loaded a wing to bring the stalling speed in approach configuration down to little more than 95 knots (little more than that of an A-6). Thus an approach speed of 115 knots is claimed for both the land and sea versions. This results in lower arresting loads than would be suffered by an aircraft which approached at faster speeds.

There are other respects in which this configuration is not so ideal for carrier operation, however. As with all deltas, the rate at which lift increases with increasing angle of attack is low, which means that quite a high angle of attack is needed for maximum lift — I would guess something like 16° to 17° for the approach. Added to a normal 3½° glide slope, this puts the landing point an uncomfortable 20° or so below the nose.

Moreover, such an aircraft would ideally launch at a high angle of attack as well — 13° or 14°, I would imagine. Obviously, this creates major difficulties in a conventional catapult launch. The sea-based *Rafale* incorporates a complex and heavy "jump" nose gear, but even this will almost surely not get it fully to optimum angle of attack (though it is claimed to cut 9 knots from the catapult end speed requirement[18]). To launch the *Rafale* from the rather feeble catapults of its existing carriers, the *Aéronavale* has adopted a variant of the ski jump idea, one first proposed some four decades ago[19]. In this scheme, a ramp is placed forward of the catapult. This is necessarily a rather small ramp (1.5° angle and about 30 feet long in the case of the French carriers), but it has beneficial effects on angle of attack as well as providing an initial rate of climb. The U.S. Navy work on ramps cited earlier also analyzed this scheme (which the authors described as CRAT, for "catapult/ramp assisted takeoff") and concluded that it provides substantial advantages for high-performance aircraft. It is claimed that in this case the ramp cuts 20 knots from the *Rafale*'s required end speed.

There seems no reason to suppose that the fundamental choice of the *Rafale*'s configuration was influenced by considerations of carrier suitability, so the *Armée de l'Air* can not well complain that the *Aéronavale*'s needs have queered its basic qualities. Because of its relatively low approach speed, the *Rafale* may perhaps pay a slightly lower overall structural weight penalty than a hotter carrier aircraft, but this is probably not enough to make a major difference. But how do the manufacturers and defense ministry avoid howls of protest from the *Armée de l'Air* over the penalties of carrier suitability?

The secret seems to appear in the difference in empty weights of the two versions. The land version is said to weigh 19,481 lb empty, as compared to 21,517 lb for the sea version. This is a difference of 2,036 lb, or 10.5% of the land version's empty weight. This is right in line with what many designers believe are the "normal" weight penalties of carrier-based operation — which suggests that through careful structural design, Dassault has been able to make an aircraft in which it is possible to build land and sea versions on the same production line with largely common tooling but with little or no weight penalty for the land version. While this is not as satisfying as finding a completely common solution for three different services, it is an interesting technical achievement which seems to bear study.

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6. Standard Aircraft Characteristics Chart, F-4C.

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Comparison of Calculated and Actual Takeoff and Landing Performance
Table 1

ASSUMPTIONS:

Takeoff speed=	1.2	*x stall speed*
Landing speed =	1.2	*x stall speed*
Installed thrust =	0.95	*x uninstalled thrust*
Ground drag =	0.05	*x gross weight*
Rotation duration =	3	*(s)*
Braking coefficient =	0.3	*x gross weight*
Free roll duration =	2	*(s)*
Wind =	0	*(kts)*
Air density =	0.00238	*(slug/cuft)*
One knot =	1.68781	*(ft/s)*
One g =	32.174	*(ft/s/s)*

Takeoff Calculations:

Aircraft	Wing area *(sqft)*	Gross Wt *(lb)*	Wing loading *(lb/sqft)*	Stall spd *(kt)*	Max Cl	Uninst Thrust *(lb)*	Inst Thrust /Wt	T/O run *(ft)*	Calc Spd @TO *(kt)*	Calc T/O run *(ft)*	Error *(ft)*	Err fract.
F-4C	530.0	59,493	112.3	155	1.39	34,000	0.54	4,260	185	4,026	-234	-0.05
F-18A	400.0	35,690	89.2	125	1.69	32,032	0.85	1,550	150	2,001	451	0.29
F-18A	400.0	39,384	98.5	131	1.69	32,032	0.77	2,010	157	2,310	300	0.15
F-18A	400.0	49,224	123.1	147	1.68	32,032	0.62	3,620	176	3,318	-302	-0.08
F-18A	400.0	35,690	89.2	125	1.69	21,216	0.56	2,300	150	2,695	395	0.17
F-18A	400.0	39,384	98.5	131	1.69	21,216	0.51	3,160	157	3,165	5	0.00
F-18A	400.0	49,224	123.1	147	1.68	21,216	0.41	5,900	176	4,725	-1,175	-0.20
F-4S	530.0	48,074	90.7	138	1.41	35,640	0.70	2,150	166	2,694	544	0.25
F-4S	530.0	52,509	99.1	144	1.41	35,640	0.64	2,610	173	3,097	487	0.19
F-4S	530.0	55,691	105.1	148	1.42	35,640	0.61	2,980	178	3,402	422	0.14
F-4S	530.0	48,074	90.7	138	1.41	23,620	0.47	3,680	166	3,752	72	0.02
F-4S	530.0	52,509	99.1	144	1.41	23,620	0.43	4,525	173	4,378	-147	-0.03
F-4S	530.0	55,691	105.1	148	1.42	23,620	0.40	5,230	178	4,856	-374	-0.07
F-14D	505.0	60,432	119.7	118	2.54	53,900	0.85	1,150	142	1,830	680	0.59
F-14D	505.0	64,492	127.7	122	2.53	53,900	0.79	1,320	146	2,017	697	0.53
F-14D	505.0	69,758	138.1	129	2.45	53,900	0.73	1,550	155	2,335	785	0.51
F-14D	505.0	60,432	119.7	118	2.54	32,666	0.51	2,040	142	2,632	592	0.29
F-14D	505.0	64,492	127.7	122	2.53	32,666	0.48	2,420	146	2,942	522	0.22
F-14D	505.0	69,758	138.1	129	2.45	32,666	0.44	3,200	155	3,470	270	0.08
A-4F	260.0	15,977	61.5	113	1.42	9,300	0.55	1,900	136	2,305	405	0.21
A-4F	260.0	18,771	72.2	123	1.41	9,300	0.47	2,640	148	3,040	400	0.15
A-4F	260.0	23,415	90.1	140	1.36	9,300	0.38	4,550	168	4,668	118	0.03
A-6E	528.9	52,003	98.3	122	1.95	18,600	0.34	3,350	146	4,016	666	0.20
A-6E	528.9	59,129	111.8	130	1.95	18,600	0.30	4,820	156	5,119	299	0.06
Viggen	495.1	37,500	75.7	115	1.69	26,015	0.66	1,310	138	2,083	773	0.59

Aircraft data from Standard Aircraft Characteristics Charts

Table 1 (Continued)

Landing Calculations:

Aircraft	Wing area *(sqft)*	Gross Wt *(lb)*	Wing loading *(lb/sqft)*	Stall spd *(kt)*	Max Cl	Lndg roll *(ft)*	Calc Lndg Spd *(kt)*	Calc Free roll *(ft)*	Calc Brake roll *(ft)*	Total Calc roll *(ft)*	Error *(ft)*	Err fract.
F-18A	400.0	26,488	66.2	108	1.68	2,490	130	434	2,062	2,497	7	0.00
F-18A	400.0	28,594	71.5	112	1.68	2,680	134	450	2,220	2,671	-9	0.00
F-18A	400.0	30,135	75.3	115	1.68	2,830	138	463	2,343	2,805	-25	-0.01
F-4S	530.0	35,495	67.0	119	1.40	2,875	143	479	2,511	2,990	115	0.04
F-4S	530.0	37,178	70.1	122	1.39	2,990	146	491	2,641	3,132	142	0.05
F-4S	530.0	38,713	73.0	124	1.40	3,100	149	499	2,729	3,228	128	0.04
F-14D	505.0	46,501	92.1	106	2.42	2,200	127	426	1,986	2,412	212	0.10
F-14D	505.0	49,842	98.7	109	2.45	2,350	131	438	2,101	2,540	190	0.08
F-14D	505.0	54,875	108.7	116	2.39	2,600	139	467	2,384	2,851	251	0.10
A-4F	260.0	12,598	48.5	101	1.40	2,305	121	406	1,800	2,206	-99	-0.04
A-4F	260.0	13,087	50.3	105	1.35	2,385	126	422	1,948	2,370	-15	-0.01
A-6E	528.9	28,595	54.1	87	2.11	1,640	104	349	1,329	1,678	38	0.02
A-6E	528.9	29,715	56.2	89	2.10	1,840	107	357	1,392	1,749	-91	-0.05
A-6E	528.9	30,556	57.8	90	2.11	1,890	108	361	1,424	1,785	-105	-0.06
Viggen	495.1	30,000	60.6	100	1.79	1,640	120	402	1,764	2,166	526	0.32

Variation of Takeoff and Landing Performance with Lift Coefficient
Table 2

ASSUMPTIONS:

Wing loading @ T/O =	100	*(lb/sqft)*
Wing loading @ landing =	65	*(lb/sqft)*
Thrust/Weight @ T/O =	0.75	*(lbf/lbm)*
T/O & Landing speed =	1.2	*x stalling speed*
Catapult stroke =	304	*(ft)*
Arresting runout =	320	*(ft)*
Braking coefficient =	0.3	*x gross weight*
Rolling drag =	0.05	*x gross weight*
Thrust/wt for reversing =	0.15	*(lbf/lbm)*
Free roll duration =	0.5	*(s)*
Wind for catapult/arrest =	15	*(kts)*
Wind for T/O & landing =	0	*(kts)*
Air density =	0.00238	*(slug/cuft)*

CALCULATED VALUES:

Max Lift Coeff	TO Stall Spd *(kts)*	TO speed *(kts)*	Cat accel *(g)*	Deck TO Run *(ft)*	Land TO Run *(ft)*	Lndg Stall Spd *(kts)*	Lndg peed *(kts)*	Arrest decel *(g)*	Deck Lndg Run *(ft)*	Land Lndg Run *(ft)*	Land Rev Run *(ft)*
1.50	140	168	2.73	1,488	1,793	113	136	1.97	1,947	2,446	1,746
1.75	130	156	2.19	1,256	1,537	105	126	1.65	1,643	2,104	1,505
2.00	122	146	1.79	1,083	1,345	98	118	1.41	1,417	1,848	1,323
2.25	115	137	1.48	949	1,196	92	111	1.22	1,243	1,648	1,181
2.50	109	130	1.24	843	1,076	88	105	1.07	1,104	1,488	1,068
2.75	104	124	1.04	756	978	84	100	0.96	992	1,356	975
3.00	99	119	0.88	685	897	80	96	0.86	898	1,247	897
3.25	95	114	0.74	625	828	77	92	0.78	820	1,154	831
3.50	92	110	0.62	574	769	74	89	0.71	753	1,074	774
3.75	89	107	0.52	529	717	72	86	0.64	695	1,005	725
4.00	86	103	0.43	491	672	69	83	0.59	645	944	682
4.25	83	100	0.35	457	633	67	81	0.55	601	891	644
4.50	81	97	0.28	428	598	65	78	0.51	562	843	610
4.75	79	95	0.22	401	566	64	76	0.47	527	801	580
5.00	77	92	0.17	377	538	62	74	0.44	496	762	552
5.25	75	90	0.12	356	512	60	73	0.41	468	727	528
5.50	73	88	0.07	336	489	59	71	0.38	442	696	505
5.75	72	86	0.03	319	468	58	69	0.36	419	667	484
6.00	70	84	0.00	303	448	57	68	0.34	398	640	465
6.25	69	82	0.00	288	430	55	67	0.32	379	616	448
6.50	67	81	0.00	275	414	54	65	0.30	361	593	432
6.75	66	79	0.00	262	399	53	64	0.28	345	572	417
7.00	65	78	0.00	251	384	52	63	0.27	330	553	403
7.25	64	77	0.00	240	371	51	62	0.25	316	534	390
7.50	63	75	0.00	230	359	51	61	0.24	303	517	378
7.75	62	74	0.00	221	347	50	60	0.23	291	502	366
8.00	61	73	0.00	212	336	49	59	0.22	279	487	356

NOTE: A zero catapult acceleration indicates that the aircraft can lift off in less than the length of the catapult stroke without any assistance

Fig. 1: SPEED VARIATION WITH MAXIMUM LIFT COEFFICIENT

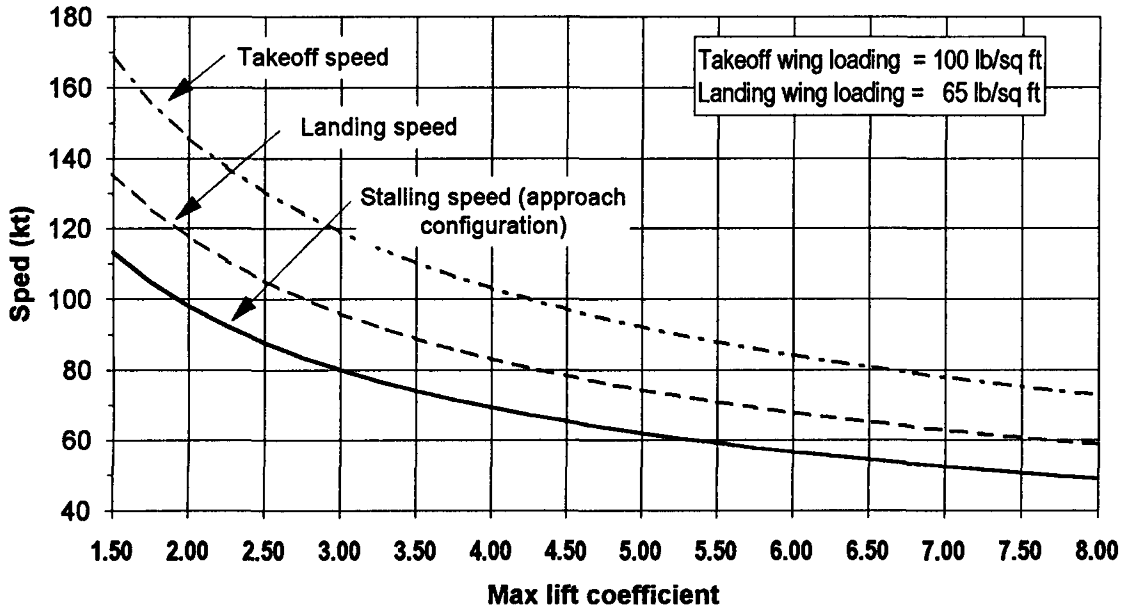
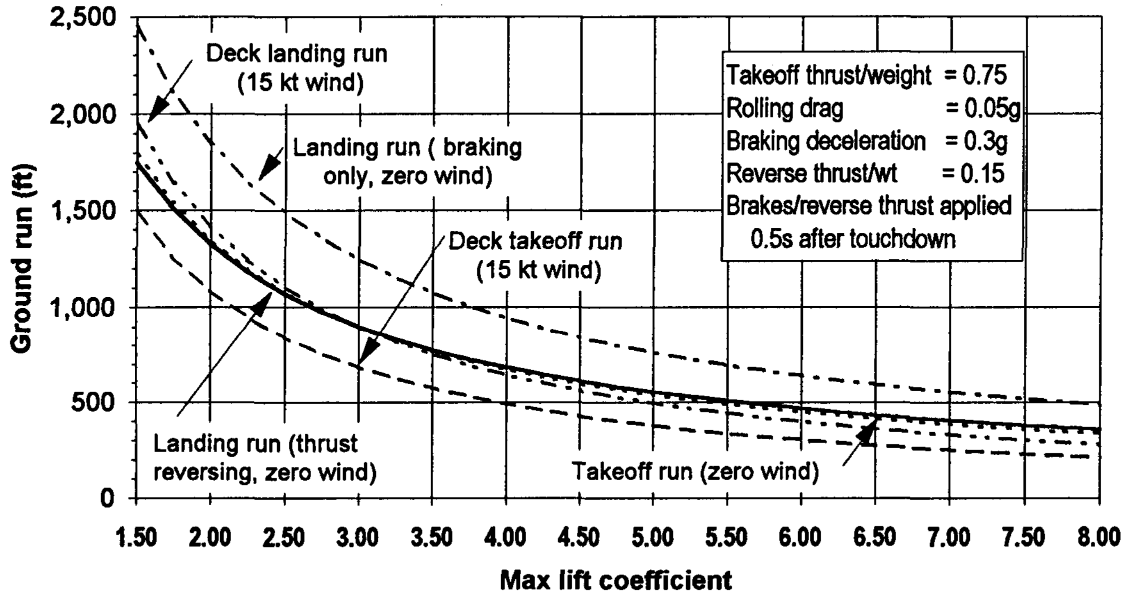


Fig. 2: VARIATION OF LANDING AND TAKEOFF RUNS WITH MAXIMUM LIFT COEFFICIENT



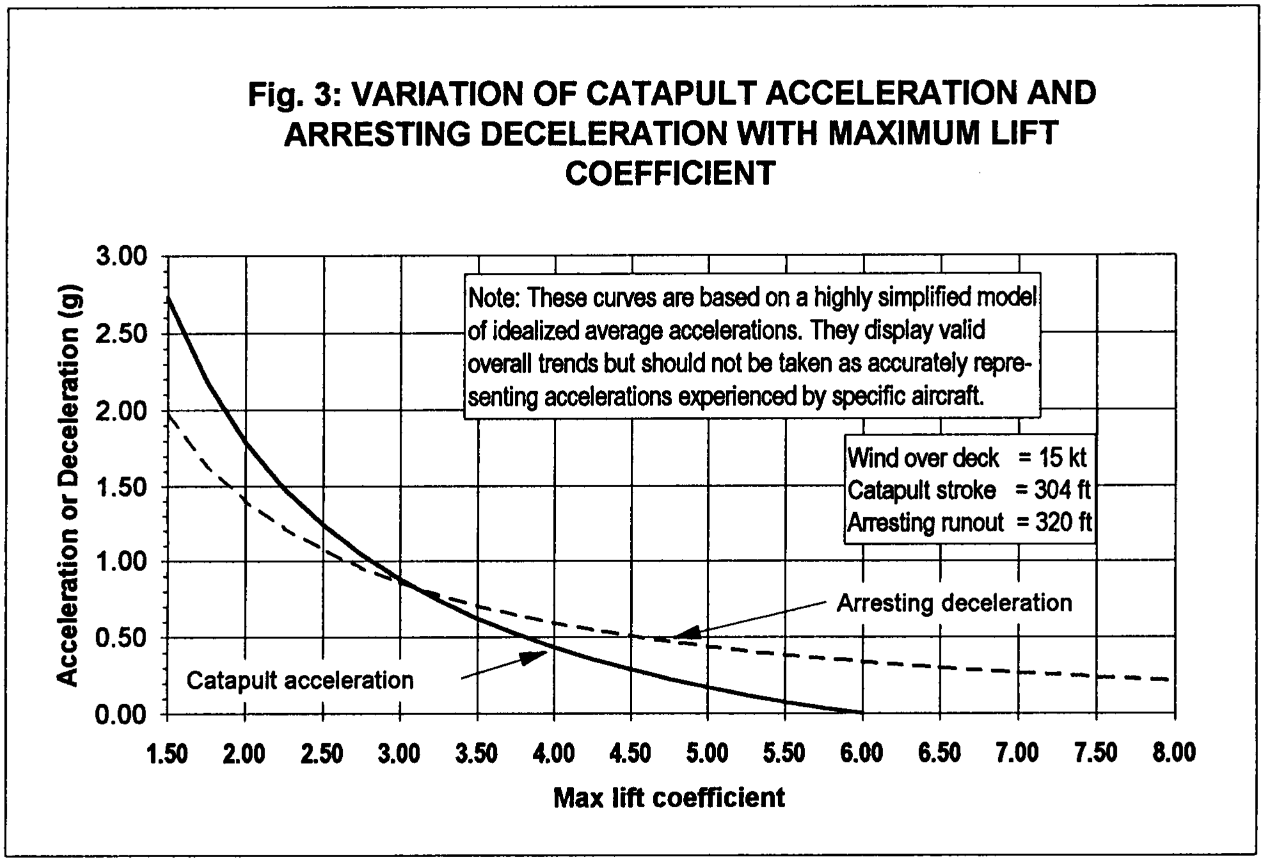


Fig. 3: VARIATION OF CATAPULT ACCELERATION AND ARRESTING DECELERATION WITH MAXIMUM LIFT COEFFICIENT
Note: These curves are based on a highly simplified model of idealized average accelerations. They display valid overall trends but should not be taken as accurately representing accelerations experienced by specific aircraft.
Wind over deck = 15 kt
Catapult stroke = 304 ft
Arresting runout = 320 ft
Arresting deceleration
Catapult acceleration
Acceleration or Deceleration (g)
3.00
2.50
2.00
1.50
1.00
0.50
0.00
1.50
2.00
2.50
3.00
3.50
4.00
4.50
5.00
5.50
6.00
6.50
7.00
7.50
8.00
Max lift coefficient

TAB A

PARAMETRIC ANALYSIS OF THE COSTS OF SEPARATE AND COMBINED AIRCRAFT PROGRAMS

Definitions:

$P(n)$	Procurement cost for the first *n* units
$R(m)$	RDT&E cost for *m* versions of the same basic aircraft
$A(m,n)$	Acquisition cost for m versions bought in total quantity n: *A(m,n) = R(m) + P(n)*
r	Rate at which procurement costs fall, on cumulative average basis, with doubling of quantity
p_1	First unit cost: *P(1)*

Take:

$$P(n,r,p_1) := p_1 \cdot n^{1+\frac{\ln(r)}{\ln(2)}}$$

Then

$$P(2\cdot n,r,p_1) = p_1 \cdot (2\cdot n)^{1+\frac{\ln(r)}{\ln(2)}} = 2\cdot r\cdot p_1 \cdot n^{1+\frac{\ln(r)}{\ln(2)}} = 2\cdot r\cdot P(n,r,p_1)$$

F-22 Program. For the F-22 we have (from the 31 December 1992 Selected Acquisition Report for the program), that *P* for *n* = 648 is $44,106 million in FY 1990 constant dollars. Using the N80 FY94CB$ Escalation Table distributed 23 Dec 1993, we find that in FY 96 constant dollars,

$$P_{648} := 44106 \cdot \frac{1.0451}{0.9018}$$

$P_{648} = 51115$ FY 1996 $million

Let us assume

$r := .825$

Then

$$p_1 := \frac{P_{648}}{648^{\left(1+\frac{\ln(r)}{\ln(2)}\right)}}$$

$p_1 = 476$ FY 1996 $million

Of course the procurement unit cost is simply

$$\frac{P_{648}}{648} = 79$$ FY 1996 $million

But it now has been announced that the number of F-22s to be produced will be not more than 442, resulting in a lower procurement cost, but higher procurement unit cost:

$$P(442, r, p_1) = 38771$$ FY 1996 $million

$$\frac{P(442, r, p_1)}{442} = 88$$ FY 1996 $million

The F-22 SAR also tells us that the total RDT&E cost is expected to be $16,482 million, again in FY 1990 constant dollars. Again we can convert this to FY 96 constant dollars:

$$R := 16482 \cdot \frac{1.0458}{0.8950}$$

$$R = 19259$$ FY 1996 $million

Note that

$$\frac{R}{p_1} = 40.5$$

Taking

$$A(n, r, p_1) := R + P(n, r, p_1)$$ Total acquisition cost

We find

$$A(648, r, p_1) = 70374$$ Total acquisition cost for 648 F-22, FY 1996 $million

$$A(442, r, p_1) = 58030$$ Total acquisition cost for 442 F-22, FY 1996 $million

$$\frac{A(648, r, p_1)}{648} = 109$$ Acquisition unit cost for 648 F-22, FY 1996 $million

$$\frac{A(442, r, p_1)}{442} = 131$$ Acquisition unit cost for 442 F-22, FY 1996 $million

There has been speculation that the F-22 program might be cut back further, to "silver bullet" status. It is not entirely clear what this might mean, but let us suppose that it would result in a total procurement of 250 aircraft. Then

$$P(250, r, p_1) = 25687$$ Total production cost for 250 F-22, FY 1996 $million

$$\frac{P(250, r, p_1)}{250} = 103$$ Production unit cost for 250 F-22, FY 1996 $million

$A(250, r, p_1) = 44946$ Total acquisition cost for 250 F-22, FY 1996 $million

$\dfrac{A(250, r, p_1)}{250} = 180$ Acquisition unit cost for 250 F-22, FY 1996 $million

It will be interesting to compare these estimates for reduced quantities with those in the 31 December 1993 SAR, when it is released.

New Programs, Separate *vs.* Combined. We shall consider an aircraft which is about 75% as costly, in equivalent quantities, as the F-22, so

$p_1 := 360$ FY 1996 $million

Now we want to consider more than one possible program. Let us define R_1 as the RDT&E cost for a single program and take it to be 40 times the first unit production cost, p_1:

$R_1 := 40 \cdot p_1$

$R_1 = 14400$ FY 1996 $million

We will assume that *R(m)* follows a power law in *m* with an exponent *c* which we will take as:

$c := 0.6$

$R(m) := R_1 \cdot m^c$

So for three programs we obtain:

$R(1) = 14400$ FY 1996 $million

$R(3) = 27838$ FY 1996 $million

Let us now suppose that we are contemplating three aircraft programs, and that (for the sake of simplicity) each will comprise one third of the total production run, N. We will consider the case in which there are three separate programs, and that in which the three are combined:

	Separate	Combined
Procurement	$3 \cdot P\left(\frac{N}{3}, r, p_1\right)$	$P(N, r, p_1)$
RDT&E	$3 \cdot R(1)$	$R(3)$

So we can take

$A_{sep}(N, r, p_1) := 3 \cdot R(1) + 3 \cdot P\left(\frac{N}{3}, r, p_1\right)$ Acquisition cost for 3 separate programs

$A_{comb}(N, r, p_1) := R(3) + P(N, r, p_1)$ Acquisition cost for 3 programs combined

Thus, for instance,

$A_{sep}(750, r, p_1) = 101525$ FY 1996 \$million

$A_{comb}(750, r, p_1) = 70835$ FY 1996 \$million

Clearly, so long as r < 1 we will always have $A_{sep}(N, r, p_1) > A_{comb}(N, r, p_1)$

It is of interest to examine the savings and the proportionate savings, and acquisition unit costs:

$S(N, r, p_1) := A_{sep}(N, r, p_1) - A_{comb}(N, r, p_1)$ Savings (absolute magnitude)

$$\sigma(N, r, p_1) := \frac{A_{sep}(N, r, p_1) - A_{comb}(N, r, p_1)}{A_{sep}(N, r, p_1)}$$ Savings as fraction of separate cost

$$AUC_{sep}(N, r, p_1) := \frac{A_{sep}(N, r, p_1)}{N}$$ Acquisition unit cost for separate programs

$$AUC_{comb}(N, r, p_1) := \frac{A_{comb}(N, r, p_1)}{N}$$ Acquisition unit cost for combined programs

We will consider a range of both procurement quantity and cost-reduction rate.

$N := 600, 675 .. 1200$ $r := 0.8, 0.825 .. 0.875$

Fig. A-1: Savings from combining programs, FY1996 \$M, for various cost reduction rates and quantities

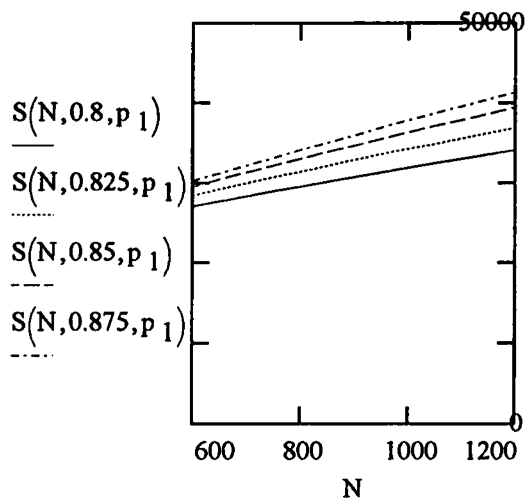


Fig. A-2: Savings from combining programs, as a fraction of the cost of separate programs, for various cost reduction rates and quantities

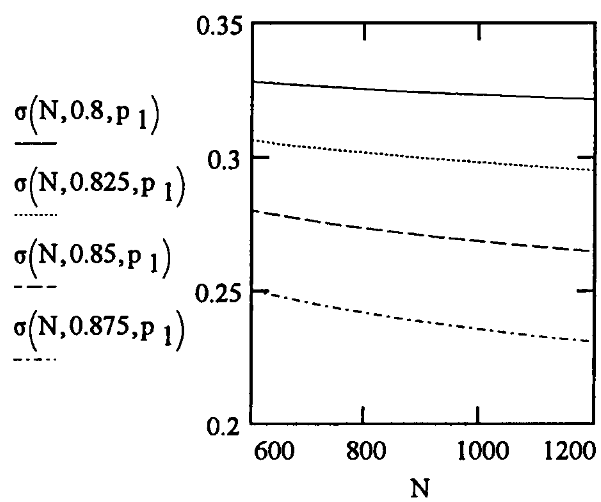


Fig. A-3: Combined program total acquisition cost in FY1996 $M, for various cost reduction rates and quantities

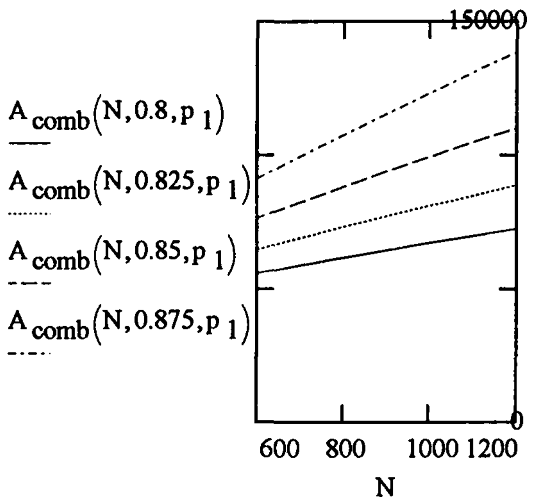
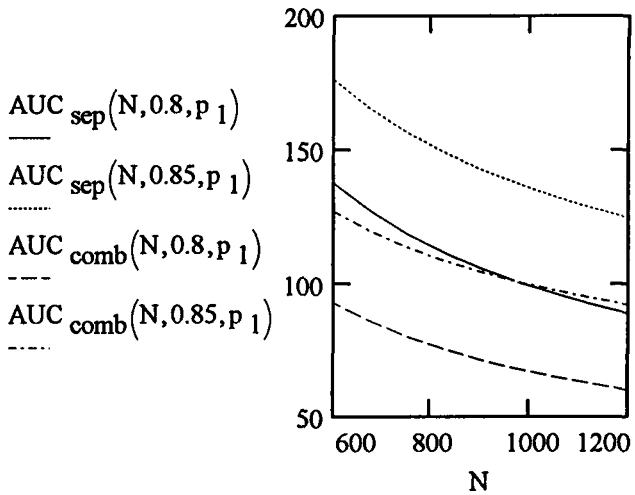


Fig. A-4: Acquisition unit cost in FY1996 $M, for various cost reduction rates and quantities, combined *vs.* separate programs



New Aircraft *vs.* More F-22s. Suppose that the Air Force, instead of buying some of the new aircraft, elected to procure additional F-22s, beyond the 442 now planned. We will suppose that some 400 more aircraft are involved, with a total of 800 for the other services. From the Air Force's point of view, the costs are

$A(842, 0.825, 476) - A(442, 0.825, 476) = 23008$ [400 more F-22s, FY 1996 $M]

$AUC_{comb}(1200, 0.825, 360) \cdot 400 = 29407$ [400 new aircraft, FY 1996 $M]

From the nation's standpoint, the grand total costs of the two alternative programs are

$A(842, 0.825, 476) + A_{comb}(800, 0.825, 360) = 153954$ [842 F-22 + 800 new aircraft]

$A(442, 0.825, 476) + A_{comb}(1200, 0.825, 360) = 146279$ [442 F-22 + 1200 new aircraft]

Thus from the Air Force's perspective, there would seem to be a savings of $6.4 billion from buying 400 more F-22s rather than participating on an equal-share basis in a new common aircraft program. On the nation's accounts, however, this course would add $7.7 billion to the sum of the costs of the two programs. Put another way, the Air Force would save $6.4 billion, but the remaining participants in the new program would have to pay an additional $6.4 billion + $7.7 billion = $14.1 billion.