OPERATIONS EVALUATION GROUP

STUDY NO. 431

MANEUVER RULES FOR DIVE BOMBING, TORPEDO BOMBING AND LEVEL-PATTERN BOMBING OF SURFACE SHIPS

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Ref: (a) ARR 18 "Analysis of U. S. Air Effort and Damage to the Japanese Fleet in the Battle for Leyte Gulf" Conf 13 Feb 1945
(b) ARR 15 "Dive Bombing Accuracy of the SBD and F4U" Conf 9 Jan 1945
(c) ARR 20 "Dive and Glide Bombing Accuracy Achieved on Marcus Island Strike of May 19 and May 20, 1944" Conf 2 Apr 1945
(d) NWC Publication "Rules for Computation of Aircraft Losses in Combat and Aerial Attack and the Assessment of Surface Damage Resulting from Aerial Attack" Conf 20 Jan 1948 (Amended 1 Jun 1948)
(e) NWC Publication "The Battle of Midway, Strategical and Tactical Analysis" Conf
(f) NWC Publication "Battle of Coral Sea, May 1 to May 11 Inclusive, 1942, Strategical and Tactical Analysis" Conf 1947
(g) ONI Publication "Japanese Story of Midway" Unclass Jun 1947
(h) ONI Review Vol. I No. 9 "The YAMATO and the MUSASHI" Conf Jul 1946
(i) ACA Forms Secret 7 Apr 1945
(j) ACA Forms Secret 24 Oct 1944
(k) ARR 32 "Review of Aircraft Torpedo Attacks in the Pacific, 1 November through 15 August, 1945" Conf 11 Dec 1945
(l) Operations Analysis Section USAF AC/AS-3 Report #3"Causes of Bombing Errors as Determined from Analysis of 8th Air Force Combat Operations" Conf 15 Jul 1943
(m) AAF Bombing Accuracy Report #1 Secret 11 Apr 1945

(n) Applied Mathematics Panel "Statistical Studies in Warfare" Conf 1946
(o) Applied Mathematics Panel "Distribution of the Percentage of Hits When Square Patterns are Dropped on Rectangular Targets" Restr Apr 1945
(p) USF 79 (A)-1 "Supplement to Umpire Rules for Fleet Exercises, (Damage Assessment Tables)" Secret 1948
(q) Office of Air Comptroller HqUSAF "Bombing Techniques at Velocities Approaching Supersonic Velocities" Restr
(r) NavShips "Ships Data, U. S. Naval Vessels" Vol. I 15 Apr 1945

I. SUMMARY

This study makes estimates of the expected numbers of hits on ships with various types of airborne weapons. These estimates are for use in formulating maneuver rules and for operational planning. The types of attack considered are dive bombing, aircraft torpedo attacks, and level pattern bombing. No estimates are given for attacks with guided missiles, or airborne rockets, nor are estimates made of the losses to the aircraft making the attack.

II. DIVE BOMBING

The accuracy to be expected in dive-bombing operations has been investigated and reported in several publications. The most common measure of bombing accuracy used is the CEP, the radius of a circle within which 50% of drops may be expected to hit.

Reference (a) presents a summary of the data available at the time of its preparation. It is shown that under ideal training conditions, a CEP of about 150 feet was obtained with World War II methods. Under ideal combat conditions, a CEP of 175 feet has been found. (See also reference (b).) For poor combat conditions with bad weather, heavy AA, inexperienced

pilots, and poor briefing, the CEP may be of the order of 300 feet, as in the case of the Marcus Island strike analyzed in reference (c). Reference (a) also showed that the percentage of hits in dive bombing of Japanese naval vessels in the battle for Leyte Gulf was consistent with a CEP of 150 to 175 feet. This is open to question, however, since the percentages of hits were based solely upon claims of Aircraft Action Reports, rather than upon actual measurements as in other analyses.

The expected percentages of hits in dive bombing set forth in reference (d), the measures now in use by the Naval War College, are drawn from the figures presented in reference (a), based upon a CEP of 235 feet, an average of operational errors. These values are:

Target	% of Hits
Large	20
Intermediate	12
Small	8
DD	4

These values are corrected for maneuvering target by the use of the following table:

REDUCTION FACTOR AGAINST TARGET WHICH MANEUVERS	
Speed of Target (kts.)	Factor to Multiply % Hits
3 to 12	1.0
12 to 25	0.8
Over 25	0.6

In order to check these figures with data from actual World War II experience, a comparison was made of the predicted numbers of hits using these tables, with the results actually obtained by dive and glide bombing in the Battle of Midway, the Battle of Coral Sea, and the attacks on the Japanese battleships, the YAMATO and the MUSASHI. Information for this comparison was drawn from

references (e) through (j). Due allowance was made for the effect of maneuvering by the targets as indicated in the narratives. The figures reported as actual hits are those which are concurred in by several reliable Japanese witnesses, as reported in the above references, and which are supported, wherever possible, by physical evidence. The results of this comparison are presented in Tables I-A through IV-A of Appendix A. The predicted number of hits is estimated to the nearest whole number.

It is seen in Appendix A that there is very close agreement between the predicted number of hits in dive-bombing attacks and the results actually achieved in these World War II actions.

It appears, consequently, that the values given in the tables above are adequate for the purpose of maneuver rules involving dive bombing.

III. AIRCRAFT TORPEDO ATTACKS

There appears to be no body of operational or training data available from which to determine aircraft torpedo firing errors, and then proceed to theoretical determinations of the expected percentages of hits. However, reference (k) presents a table (reproduced as Table I) of the aircraft torpedo attacks in the Pacific during World War II, and the results of such attacks, based upon claims made in the Aircraft Action Reports.

Despite careful collation and evaluation of pilots' claims during preparation of action reports, some overstatement of combat results is inevitable. The differences between claimed and actual hits during dive bombing, shown in Table IV-A of Appendix A, offer an example.

The only basis for the preparation of tables of expected hits, therefore, appears to lie in the comparison of claimed hits and actual hits as determined from enemy records. From such comparisons, a "reliability factor" for claims can be estimated, and in the absence of better information, this "factor" applied to the percentages shown in Table I.

TABLE I

Survey of Torpedo Accuracy Trends During World War II

Period	CV and CVL No. of Attacks	CV and CVL % Hits	BB No. of Attacks	BB % Hits	CA and CL No. of Attacks	CA and CL % Hits	DD and DE No. of Attacks	DD and DE % Hits	M/V No. of Attacks	M/V % Hits
A. Accuracy Against Maneuvering Targets										
1942 Nov '43	61	31	8	38	69	17	5	20	4	50
May '44 June '44	0	-	0	-	54	17	38	8	0	-
Aug '45	52	60	127	39	83	33	53	19	24	25
B. Accuracy Against Targets Underway										
1942 Nov '43	5	80	4	50	10	50	0	-	8	38
May '44 June '44	0	-	0	-	55	22	20	40	11	9
Aug '45	16	56	33	64	27	56	48	48	120	44
C. Accuracy Against Targets Not Underway										
1942 Nov '43	0	-	6	83	5	40	4	50	26	46
May '44 June '44	0	-	0	-	3	100	0	-	61	31
Aug '45	1	100	0	-	35	83	11	73	170	49

Accurate data as to aircraft torpedo hits actually obtained on naval vessels are available for the Battle of Coral Sea and the Battle of Midway. However, in view of the vast improvements in U. S. aircraft torpedo characteristics since those battles, when the torpedoes were erratic and no faster than the targets, it has been deemed unwise to use this information to evaluate more modern attacks.

The only actions in which the improved aircraft torpedo equipment was employed, and for which a careful assessment of actual hits has been made from enemy sources, are the attacks on the battleships YAMATO and MUSASHI during the Battle for Leyte Gulf, and the attacks on the YAMATO on April 7, 1945. References (h), (i), and (j) report these attacks and their assessment.

A summary of the data available is given in Table II.

TABLE II

A/C Torpedo Attacks on YAMATO Class BB's

Action	Torpedoes released	Hits Claimed (ACA Reports)		Actual Hits	
		No.	%	No.	%
Leyte Gulf	42	21	50	10	24
7 April 1945	49	31	63	14	29

It appears from an examination of Table II that the best estimate of the "reliability factor" for claimed hits is about .5. In the absence of other authoritative data as to the actual numbers of hits obtained in other actions, this factor has been applied to the figures given in Table I, and Table III prepared for use in maneuver rules.

TABLE III

% OF HITS WITH AIRCRAFT TORPEDOES ON SURFACE CRAFT TARGETS

Target	% Torpedo Hits		
	At Anchor	Underway	Maneuvering
BB,CV,CVB	45	30	25
CVL, CA	40	25	20
CVE,CL	35	20	15
DD	30	15	10
Aux, M/V	40	25	20

IV. LEVEL PATTERN BOMBING

Very little information is available as to the verified performance of high-level bombing of ship targets. The bulk of data on the accuracy of operational level bombing is derived from the strategic bombing of land targets. Such reported accuracies vary widely with the theater, period and the current tactics in use. Reference (l), for instance, includes an analysis of 89 pattern-bombing missions by the Eighth Air Force between May 1944 and February 1945, during which a mean radial error (of the MPI of the pattern) of 1210 feet from an average altitude of 23,300 feet was obtained, corresponding to a CEP of about 50 mils. Reference (m) analyzes 258 pattern bombing releases by the VIII Bomber Command during April and May 1944. These releases, from altitudes ranging from 12,000 feet to 27,000 feet had CEP's of the pattern center of from 38 to 46 mils. Reference (m) also includes some data on B-29 releases over Japan. During one set of missions with an average bombing altitude of 24,500 feet, 47% of bombs dropped were within 2000 feet of the aiming points. During another set, with an average altitude of 20,000 feet and more favorable wind conditions, 48% of bombs were within 1000 feet of the aiming points. Reference (n) points out that a CEP of 25 mils was assumed for early Applied Mathematics Panel work concerning level bombing of ships, but that "These values were chosen early in the war before more realistic (larger) estimates came to hand".

To summarize, it appears that operational, visual high-level pattern bombing during World War II resulted in a CEP of the pattern center of from 40 to 60 bombing mils (feet per thousand feet of altitude). It is not strictly true that bombing errors vary directly as the bombing altitude, but it is nearly enough so for the purposes of this study. Also, there may be some error introduced by application of measures found in bombing of land targets to the situation where ships at sea are the targets. Ships may be easier to hit than comparable land targets because they provide a better point of aim; on the other hand, they may be harder to hit because of motion and maneuver. However, in this analysis, 30-60 mil errors will be accepted as characteristic.

The situation to which the derived rules apply is that in which a formation of from 3 to 18 bombers makes a run on the target ship, each plane dropping its bombs in train, with the trains of bombs forming a roughly rectangular pattern whose center is aimed at the target.

The CEP of the pattern center may cause the entire pattern to miss the target ship, or cause the pattern to cover only a portion of the target. A small, dense pattern may produce many hits if it covers the target, but may have a high probability of missing the target entirely. A large, loose pattern may have a high probability of covering the target, or a neighboring ship, but the percentage of bombs hitting will be small.

It will be seen, therefore, that the assessment of the results of high-level pattern bombing of ship targets, in terms of percentage of bombs hitting, must consider the following factors:

a. CEP of pattern center (bombing altitude)
b. Size of the bombing pattern
c. Size of target ships
d. Spacing of ships in formation

Reference (o) discusses a method of determining the probabilities of hits under these conditions, and suggests a method of introducing an element of chance into the assignment of hits to the target ship. A set of tables of expected percentages of hits has been computed, using a CEP of 30-60 mils, by a variation of the method described in reference (o) (see Appendix C) and these are presented in Appendix B.

In using the tables, it will be necessary to specify:

a. Bombing altitude.
b. Number of planes and bombs for each pattern.
c. Pattern size.
d. Target ship for each pattern.

Normally not more than about 18 planes can be successfully maneuvered to drop a single pattern. It is suggested that the following may be considered usual relationships between number of planes dropping at once and pattern size:

2 - 4 planes - 500 ft. pattern
5 - 9 planes - 1000 ft. pattern
10 - 18 planes - 1500 ft. pattern

The assessment of percentage of hits from a level-bombing attack would be made as follows:

a. The line of the proper table for pattern size, target type, and altitude is selected.

b. A single die is rolled.

c. The percentage of hits on the target ship from bombs in the pattern would be as indicated in the column corresponding to the number rolled.

d. Fractional hits should not be assessed directly. Chance tables, such as are given in reference (p), should be used to determine whether a hit is scored, assigning a percentage chance of hit equal to the fractional hit.

e. The above steps are repeated for each pattern dropped during the attack.

Because of the paucity of data it is difficult to check these tables with any degree of precision. Table IV on the Battle of Midway gives at least a qualitative check.

(LO)2065-50
17 October 1950

TABLE IV

BATTLE OF MIDWAY
AMERICAN LEVEL-BOMBING ATTACKS ON JAPANESE VESSELS

Attack Time	Target	No. Planes /No. Bombs	Assumed Pattern Diam.	Bombing Altitude	Predicted Hits, Tables Appendix B	Actual Hits	Claimed Hits
0809/4 June	CV	14/112	1500 ft.	20,000 ft.	0	0	5
1820/4 June	CA	4/28	500 ft.	25,000 ft.	0	0	1
1835/4 June	CA	3/4	500 ft.	4,000 ft.	1	0	1
1840/4 June	CA	2/8	500 ft.	10,000 ft.	0	0	3
0815/5 June	CL	8/39	1000 ft.	20,000 ft.	0	0	1
1635/5 June	DD	3/24	500 ft.	15,000 ft.	0	0	2
1635/5 June	DD	4/36	500 ft.	15,000 ft.	0	0	2
1840/4 June	DD	3/4	500 ft.	4,000 ft.	0	0	1
1639/3 June	Aux.	3/12	500 ft.	10,000 ft.	0	0	
1639/3 June	"	3/12	500 ft.	10,000 ft.	0	0	5
1639/3 June	"	3/12	500 ft.	10,000 ft.	0	0	
				Total	1	0	20

Current Air Force analyses indicate that the errors to be expected in level bombing by radar are about twice those for visual bombing. Therefore, in using the tables for radar bombing, enter the table at an altitude which is twice that at which the attack takes place.

The assumption of a 30-60 mil CEP is based upon aircraft speeds of about 250 knots. Reference (q) indicates that the errors increase in direct proportion to the speed, up to about 500 knots. In the use of the tables, therefore, increasing the bomber speed above 250 knots may be considered equivalent to increasing the bombing altitude proportionately.

Similarly, when new information indicates significant increases in the accuracy of level bombing, the decrease in the CEP may be considered equivalent to bombing from a proportionately lower altitude, and the use of the tables adjusted accordingly.

The use of guided bombs, such as AZON and RAZON, is limited to individually dropped, individually controlled bombs at present. The effect of such control is, in general, to halve the CEP. In assessing the percentages of hits for such bombs, therefore, it is suggested that Table 21 of reference (p) be used, for altitudes one half of the bombing altitudes.

The question is now considered as to hits which may be obtained on ships other than the target ship. It will be noted that the maximum pattern size considered is a 1500-foot square, so that no more than one ship is considered hit so long as ship spacings are 1500 feet (500 yards) or more. However, in tight formations, a pattern missing the target ship has a chance of producing hits on a neighboring ship. This chance is small and may be disregarded for problem purposes. A method for assessing such hits has been developed and may be applied as follows:

a. If the expected percentage of hits on the target ship is zero, hits on a neighboring ship will be assessed according to Table V.

TABLE V

DETERMINATION OF HITS ON SHIPS
ADJACENT TO TARGET SHIP

Ship Spacing	Pattern Size		
	500 ft.	1000 ft.	1500 ft.
Assess hits on neighboring ship if second roll of die yields:			
500 yards	-	3	any number
1000 yards	-	-	3
over 1000 yards	-	-	-

b. To determine the percentage of hits on the neighboring ship, after a favorable second roll as above, continue rolling the die until some number comes up for which some hits are credited in the proper line of the Tables in Appendix B - then assess the corresponding percentage of hits.

Submitted by:

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Approved by:

Edward S. Lamar

E. S. LAMAR
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APPENDIX A

TABLE I-A

THE BATTLE OF CORAL SEA (1 - 11 MAY 1942)

Attack Time	Bombs Dropped in Attack	Predicted No. of Hits (reference (d))	Actual No. of Hits
American Dive and Glide Bombing of Japanese Vessels			
Target: CV			
1050/7 May	13	3	0
1117/7 May	34	7	13
1049/8 May	24	4	2
1140/8 May	4	1	1
	Total	15	16
Japanese Attacks on American Vessels			
Target: CV			
1109/8 May	14	3	2
1113/8 May	15	3	1
	Total	6	3
Target: CA			
1113/8 May	5	1	0
Target: DD and AO			
1131/6 May	DD 9 (?)	0	3
1131/6 May	AO 15 (?)	2	7
	Total	2	10

TABLE II-A

THE BATTLE OF MIDWAY (27 MAY TO 9 JUNE 1942)

Attack Time	Bombs Dropped in Attack	Predicted No. of Hits (reference (d)	Actual No. of Hits
American Dive and Glide Bombing of Japanese Vessels			
Target: CV (Maneuvering)			
0755/4 June	16	2	0
0806/4 June	16	2	0
1022/4 June	65	7	9
1024/4 June	9	1	0
1701/4 June	22	2	4
	Total	14	13
Target: BB (Maneuvering)			
0820/4 June	12	1	0
0827/4 June	11	1	0
1024/4 June	2	0	0
1720/4 June	14	1	0
1700/4 June	2	0	0
	Total	3	0
Target: CA			
1024/4 June	2	0	0
0950/6 June	12	2	5
1130/6 June	31	6	3
1445/6 June	23	4	4
	Total	12	12
Target: DD (Maneuvering)			
1750/5 June	32	1	0
1810/5 June	11	0	0
	Total	1	0

TABLE III-A

AMERICAN DIVE BOMBING ATTACKS ON THE JAPANESE BATTLESHIPS MUSASHI AND YAMATO 24 OCTOBER 1944

Bombs Dropped per Attack	Claimed Hits (ACA Forms)	Predicted Hits (reference (d))	Actual Hits
7	4	1	
29	3 - 6	6	
6	3	1	
5	3	1	
4	2	1	
12	7	2	
6	2	1	
18	11	4	
Total	35 - 38	17	(ONI)19

TABLE IV-A

AMERICAN DIVE BOMBING ATTACKS ON THE BATTLESHIP YAMATO 7 APRIL 1945

Bombs Dropped per Attack	Claimed Hits (ACA Forms)	Predicted Hits (reference (d))	Actual Hits
4	0	1	0
10	4	2	4
8	3	2	No Information
2	0	0	"
1	1	0	"
24	8	5	"
3	1	1	"
27	20	6	"
Total	37	17	

APPENDIX B

PERCENTAGE OF HITS ON TARGET SHIP
LEVEL BOMBING

1500-FT. PATTERN

Bombing Altitude (in ft.)	1	2	3	4	5	6	Mean
Target: BB (Large)							
5000 - 9000	1.5	3.2	3.4	3.4	3.4	3.4	3.1
10,000 - 19,000	0.2	1.1	2.3	3.1	3.4	3.4	2.3
20,000 - 29,000	0.0	0.0	0.0	0.6	2.0	3.2	1.0
30,000 - 39,000	0.0	0.0	0.0	0.0	0.9	2.6	0.6
40,000 and over	0.0	0.0	0.0	0.0	0.0	1.7	0.3
Target: CVB							
5000 - 9000	2.8	5.7	6.0	6.0	6.0	6.0	5.4
10,000 - 19,000	0.3	1.9	4.1	5.5	6.0	6.0	3.8
20,000 - 29,000	0.0	0.0	0.0	1.2	3.8	5.6	1.8
30,000 - 39,000	0.0	0.0	0.0	0.0	1.2	4.2	0.9
40,000 and over	0.0	0.0	0.0	0.0	0.0	3.0	0.5
Target: CA (Intermediate)							
5000 - 9000	0.9	1.8	1.8	1.8	1.8	1.8	1.6
10,000 - 19,000	0.0	0.5	1.4	1.7	1.8	1.8	1.2
20,000 - 29,000	0.0	0.0	0.0	0.3	1.1	1.7	0.5
30,000 - 39,000	0.0	0.0	0.0	0.0	0.5	1.4	0.3
40,000 and over	0.0	0.0	0.0	0.0	0.0	0.9	0.2

PERCENTAGE OF HITS ON TARGET SHIP
LEVEL BOMBING

1500-FT. PATTERN

Bombing Altitude (in ft.)	Role of a Single Die 1	2	3	4	5	6	Mean
Target: CL							
5000 - 9000	0.7	1.4	1.4	1.4	1.4	1.4	1.3
10,000 - 19,000	0.0	0.4	1.1	1.4	1.4	1.4	0.9
20,000 - 29,000	0.0	0.0	0.0	0.2	0.9	1.4	0.4
30,000 - 39,000	0.0	0.0	0.0	0.0	0.3	1.0	0.2
40,000 and over	0.0	0.0	0.0	0.0	0.0	0.7	0.1
Target: MV (Small)							
5000 - 9000	0.6	1.2	1.2	1.2	1.2	1.2	1.1
10,000 - 19,000	0.0	0.3	1.0	1.2	1.2	1.2	0.8
20,000 - 29,000	0.0	0.0	0.0	0.2	0.8	1.2	0.4
30,000 - 39,000	0.0	0.0	0.0	0.0	0.3	0.9	0.2
40,000 and over	0.0	0.0	0.0	0.0	0.0	0.6	0.1
Target: DD							
5000 - 9000	0.3	0.6	0.6	0.6	0.6	0.6	0.6
10,000 - 19,000	0.0	0.2	0.5	0.6	0.6	0.6	0.5
20,000 - 29,000	0.0	0.0	0.0	0.0	0.3	0.6	0.2
30,000 - 39,000	0.0	0.0	0.0	0.0	0.2	0.5	0.1
40,000 and over	0.0	0.0	0.0	0.0	0.0	0.3	0.0

PERCENTAGE OF HITS ON TARGET SHIP
LEVEL BOMBING

1000-FT. PATTERN

Bombing Altitude (in ft.)	Roll of a Single Die 1	2	3	4	5	6	Mean
Target: BB(Large)							
5000 - 9000	2.0	4.7	6.0	7.2	7.6	7.6	5.7
10,000 - 19,000	0.0	0.0	1.3	3.9	5.9	7.1	3.0
20,000 - 29,000	0.0	0.0	0.0	0.0	1.4	5.3	1.1
30,000 and over	0.0	0.0	0.0	0.0	0.0	3.8	0.6
Target: CVB							
5000 - 9000	3.5	8.4	10.5	12.5	13.5	13.5	10.3
10,000 - 19,000	0.0	0.2	2.8	7.0	10.2	12.5	5.5
20,000 - 29,000	0.0	0.0	0.0	0.0	2.3	9.0	2.0
30,000 and over	0.0	0.0	0.0	0.0	0.0	6.3	1.0
Target: CA (Intermediate)							
5000 - 9000	1.1	2.8	3.7	4.0	4.0	4.0	3.3
10,000 - 19,000	0.0	0.0	0.7	2.2	3.4	4.0	1.7
20,000 - 29,000	0.0	0.0	0.0	0.0	0.7	2.7	0.6
30,000 and over	0.0	0.0	0.0	0.0	0.0	2.0	0.3

PERCENTAGE OF HITS ON TARGET SHIP
LEVEL BOMBING

1000-FT. PATTERN

Bombing Altitude (in ft.)	Roll of a Single Die 1	2	3	4	5	6	Mean
Target: CL							
5000 - 9000	0.8	2.3	2.9	3.1	3.1	3.1	2.5
10,000 - 19,000	0.0	0.0	0.6	1.6	2.7	3.1	1.3
20,000 - 29,000	0.0	0.0	0.0	0.0	0.5	2.1	0.4
30,000 and over	0.0	0.0	0.0	0.0	0.0	1.6	0.3
Target: DD							
5000 - 9000	0.4	1.1	1.4	1.4	1.4	1.4	1.2
10,000 - 19,000	0.0	0.0	0.2	0.8	1.3	1.4	0.6
20,000 - 29,000	0.0	0.0	0.0	0.0	0.2	0.9	0.2
30,000 and over	0.0	0.0	0.0	0.0	0.0	0.7	0.1
Target: MV							
5000 - 9000	0.7	2.0	2.5	2.7	2.7	2.7	2.2
10,000 - 19,000	0.0	0.0	0.5	1.4	2.3	2.7	1.2
20,000 - 29,000	0.0	0.0	0.0	0.0	0.4	0.8	0.2
30,000 and over	0.0	0.0	0.0	0.0	0.0	0.6	0.1

PERCENTAGE OF HITS ON TARGETS SHIP
LEVEL BOMBING

500-FT. PATTERN

Bombing Altitude (in ft.)	Roll of a Single Die 1	2	3	4	5	6	Mean
Target BB (Large)							
5000 - 9000	0.0	2.1	6.4	14.2	19.7	23.8	10.9
10,000 - 19,000	0.0	0.0	0.0	1.4	7.6	18.2	4.5
20,000 and over	0.0	0.0	0.0	0.0	0.0	12.2	2.0
Target: CVB							
5000 - 9000	0.0	1.6	11.9	24.8	33.0	39.0	18.5
10,000 - 19,000	0.0	0.0	0.0	2.7	14.6	32.7	8.3
20,000 and over	0.0	0.0	0.0	0.0	0.0	21.6	3.6
CA (Intermediate)							
5000 - 9000	0.0	0.3	3.2	7.7	10.7	12.7	6.8
10,000 - 19,000	0.0	0.0	0.0	0.0	2.7	9.6	2.1
20,000 and over	0.0	0.0	0.0	0.0	0.0	6.8	1.1
Target: CL							
5000 - 9000	0.0	0.0	2.3	6.1	8.7	10.8	4.7
10,000 - 19,000	0.0	0.0	0.0	0.0	2.1	8.9	1.8
20,000 and over	0.0	0.0	0.0	0.0	0.0	5.9	1.0

PERCENTAGE OF HITS ON TARGET SHIP
LEVEL BOMBING

500-FT. PATTERN

Bombing Altitude (in ft.)	Roll of a Single Die 1	2	3	4	5	6	Mean
Target: DD							
5000 - 9000	0.0	0.0	1.0	2.9	4.6	5.3	2.3
10,000 - 19,000	0.0	0.0	0.0	0.0	0.7	3.6	0.7
20,000 and over	0.0	0.0	0.0	0.0	0.0	2.8	0.5
Target: MV (Small)							
5000 - 9000	0.0	0.0	2.0	5.2	7.5	9.2	4.0
10,000 - 19,000	0.0	0.0	0.0	0.0	1.8	7.6	1.6
20,000 and over	0.0	0.0	0.0	0.0	0.0	5.1	.9

APPENDIX C

1. Computation of Level Bombing Tables

The major part of the problem is to calculate the probability that at least a certain percentage of the target area will be covered when square patterns are dropped on rectangular targets. The aiming point error is assumed to be normal, and is specified; the sides of the target and pattern are assumed to be parallel; and the bombs are assumed to be uniformly distributed within the pattern.

Under these assumptions, the question of finding the probability that at least a certain percentage of the target area will be covered by the pattern may be reformulated as follows: what is the probability that the center of the pattern will fall within an area whose boundary is the locus of points on which the pattern center must fall in order to give exactly the required percentage of coverage? Conversion to a percentage of hits to be expected with a given coverage is accomplished by multiplying the percentage of the target covered by the pattern by the probability that the ship will receive a hit if it is fully covered by the pattern. Under the assumption of a uniform distribution of the bombs within the pattern, this last probability is merely the ratio of the area of the ship to the area of the pattern.

Determination of the probability that the center of the pattern will fall within a specified area about the point of aim, assuming a circular normal distribution of the mean point of impact about the point of aim, is readily seen to be a problem which can be handled through the use of circular probability paper.*

Except in the case of carrier vessels, the assumption of a rectangular shape whose dimensions are the maximum length and maximum width of the ship types would be misleading, in that the results found would indicate too high a percentage of hits. Therefore, it was assumed that

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* For a discussion of the use of circular probability paper, see OEG Report 57, Appendix (i).

the ship shapes closely approximated ellipses, and the width of the ship was adjusted to give a rectangle of equal area to that of an ellipse whose major and minor diameters were the length and maximum width of the ships respectively.

Under the assumptions previously stated, it can be seen that the locus of pattern center for 0% coverage is a rectangle of dimensions (width of target plus width of pattern) (length of target plus length of pattern) centered about the point of aim (see Figure 2C). The assumption of a square pattern eliminates the necessity for computing two sets of values to correspond to beam and bow or stern attacks. The locus of pattern center for 100% coverage is a rectangle of dimensions (width of pattern minus width of target) (length of pattern minus length of target). It is demonstrated in reference (o) that the locus of pattern centers for constant percentages of coverage are made up (for a single quadrant, by symmetry) of a horizontal line, a hyperbolic arc, and a vertical line, whose distances from the point of aim are functions of the pattern dimensions, the target dimensions, and the percentage of coverage under consideration. For the computations in this paper, the locus of pattern centers was approximated by determining the horizontal and vertical portions of the locus, and then computing just one point on the hyperbolic arc, and drawing a smooth curve between the end points of the lines, through the point computed on the arc. The actual dimensions of these loci are determined from the probability paper used and the CEP assumed for the MPI, the center of the ship.

The probability of at least $\underline{h}$ coverage is then plotted against $\underline{h}$ for each of the ship types, pattern sizes, and CEP's assumed, so that the probability of at least $\underline{h}$ coverage is increasing downward on the right side of the graph (see Figure 3C). The corresponding increasing values on the left side of the graph are then the probabilities of not more than $\underline{h}$ coverage.

In order to use a die to introduce chance into expected percentage of hits, we use the curves determined as above to obtain tables of sextiles. The values in these tables indicate that in one-sixth of a large number of cases, the coverage on the ship will be less than h_1%; one sixth of the time, between h_1% and h_2%; etc. The mean of the limits in each of the sextiles has been selected as an approximation in the preparation of the tables of Appendix B. The figures in the table are then multiplied

by the appropriate ratio of ship area to pattern area to provide the expected percentage of hits. The CEP's in feet were converted to equivalent altitudes by assuming mil values of the CEP of 30 to 60 mils.

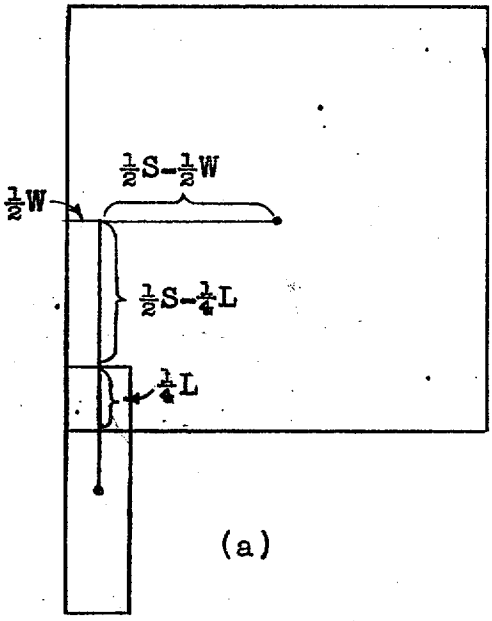
As an example, consider the problem of determining the expected percentage of hits on a CV when a 1500' pattern is dropped from an altitude corresponding to a CEP of 600 feet. Let L be the length of the target; W, the width; and S, the size of the pattern. Computations will be carried out for only one quadrant, because of the symmetry of the diagram.

In order to obtain at least 100% coverage, the center of the pattern must fall within a rectangle of dimensions (S - L)(S - W). For at least 0% coverage, the pattern center must fall within a rectangle of dimensions (S + L) (S + W).

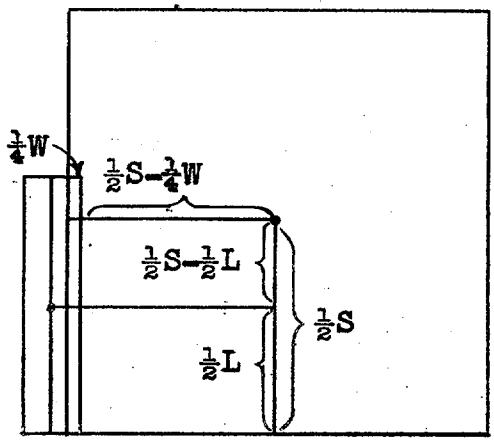
To obtain the locus of points on which the pattern center must fall to give 25% coverage on the ship, we examine Figure 1C. On diagram (a) we see the condition under which the pattern center may be displaced vertically to its greatest extent consistent with 25% coverage on the ship. The amount of this vertical displacement from the center of the ship is $\frac{1}{2}$ S + $\frac{1}{4}$ L, and this vertical displacement of the pattern center, together with any horizontal displacement up to a value of $\frac{1}{2}$ S - $\frac{1}{2}$ W gives a constant 25% coverage on the ship. This gives us sufficient information to draw the horizontal section of the locus, given pattern and target dimensions.

On diagram (b) we see that the greatest horizontal displacement consistent with 25% coverage is $\frac{1}{2}$ S + $\frac{1}{4}$ W and that this may be accompanied by a vertical displacement of $\frac{1}{2}$ S - $\frac{1}{2}$ L.

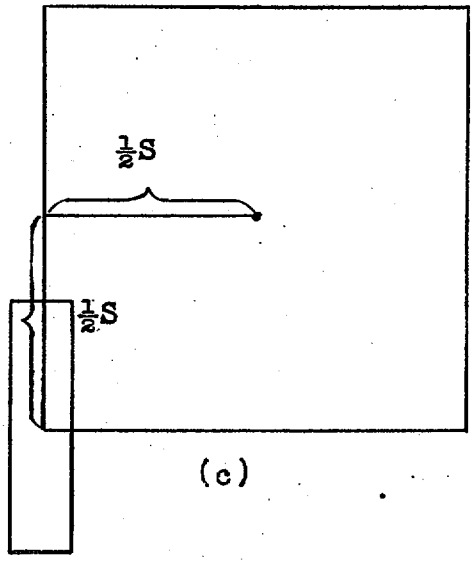
In order to determine one point on the hyperbolic section of the locus, we now examine diagram (c). It is clear that horizontal and vertical displacements of the pattern center from the target center equal to $\frac{1}{2}$ S also result in 25% coverage of the target. Similar general formulae have been worked out for 50% and 75% coverage of the target.



(a)



(b)



(c)

S = Pattern Size
L = Ship Length
W = Equivalent Ship Width
(see p. 24)

FIG. 1C PATTERN POSITIONS FOR 25% TARGET COVERAGE

Examining now the specific problem of determining hits on a CV; we first refer to the probability paper to determine the length of one CEP. In the case of the paper used for these computations, this was 2.75". Therefore, all dimensions will be converted to CEP's, and then laid out on graph paper so that a dimension of one CEP will be 2.75" in length.

The following dimensions of a CV were taken from reference (r):

L - 900'
S is assumed as 1500';
W - 150' CEP - 600'

The dimensions of the locus of 0% coverage in one quadrant are:

$\frac{1}{2}$ (S + L) x $\frac{1}{2}$ (S + W)

Vertical $\frac{\frac{1}{2}(1500 + 900)}{600} \times 2.75" = 5.50"$

Horizontal $\frac{\frac{1}{2}(1500 + 150)}{600} \times 2.75" = 3.80"$

The dimensions of the locus of 100% coverage are:

Vertical $\frac{\frac{1}{2}(1500 - 900)}{600} \times 2.75" = 1.38"$

Horizontal $\frac{\frac{1}{2}(1500 - 150)}{600} \times 2.75" = 3.10"$

For the locus of pattern center corresponding to 25% coverage, we have:

Maximum Vertical $\frac{1}{2}S + \frac{1}{4}L$

$\frac{1}{2}(S + \frac{1}{2}L) = \frac{\frac{1}{2}(1500 + 450)}{600} \times 2.75" = 4.45"$

with Horizontal

$\frac{1}{2}(S - W) = \frac{\frac{1}{2}(1500 - 150)}{600} \times 2.75" = 3.10"$

Maximum Horizontal

$$\tfrac{1}{2}\,(S + \tfrac{1}{2}\,W) = \frac{\tfrac{1}{2}\,(1500 + 75)}{600} \times 2.75" = 3.60"$$

with Vertical

$$\tfrac{1}{2}\,(S - L) = \frac{\tfrac{1}{2}\,(1500 - 900)}{600} \times 2.75" = 1.38"$$

Point on hyperbola

$$\tfrac{1}{2}\,S = \frac{\tfrac{1}{2} \times 1500}{600} \times 2.75" = 3.45"$$

The above loci have been plotted in Figure 2C.

This diagram, plus loci for 50% and 75% coverage, drawn on thin paper, is placed on the probability paper so that the 0 point is at the center of the field of points. The number of points covered by each of the areas enclosed by the loci, times four, divided by 1000 (Number of points on paper), is the probability that the pattern center will fall within that area, and thus will be the probability that at least that percentage of the target corresponding to the locus used will be covered by the pattern, under the assumptions given at the beginning of this discussion. These probabilities have been plotted against the percent coverage in Figure 3C. The left-hand scale is the probability of not more than h coverage, and it is with this that we will be concerned during the remainder of the discussion.

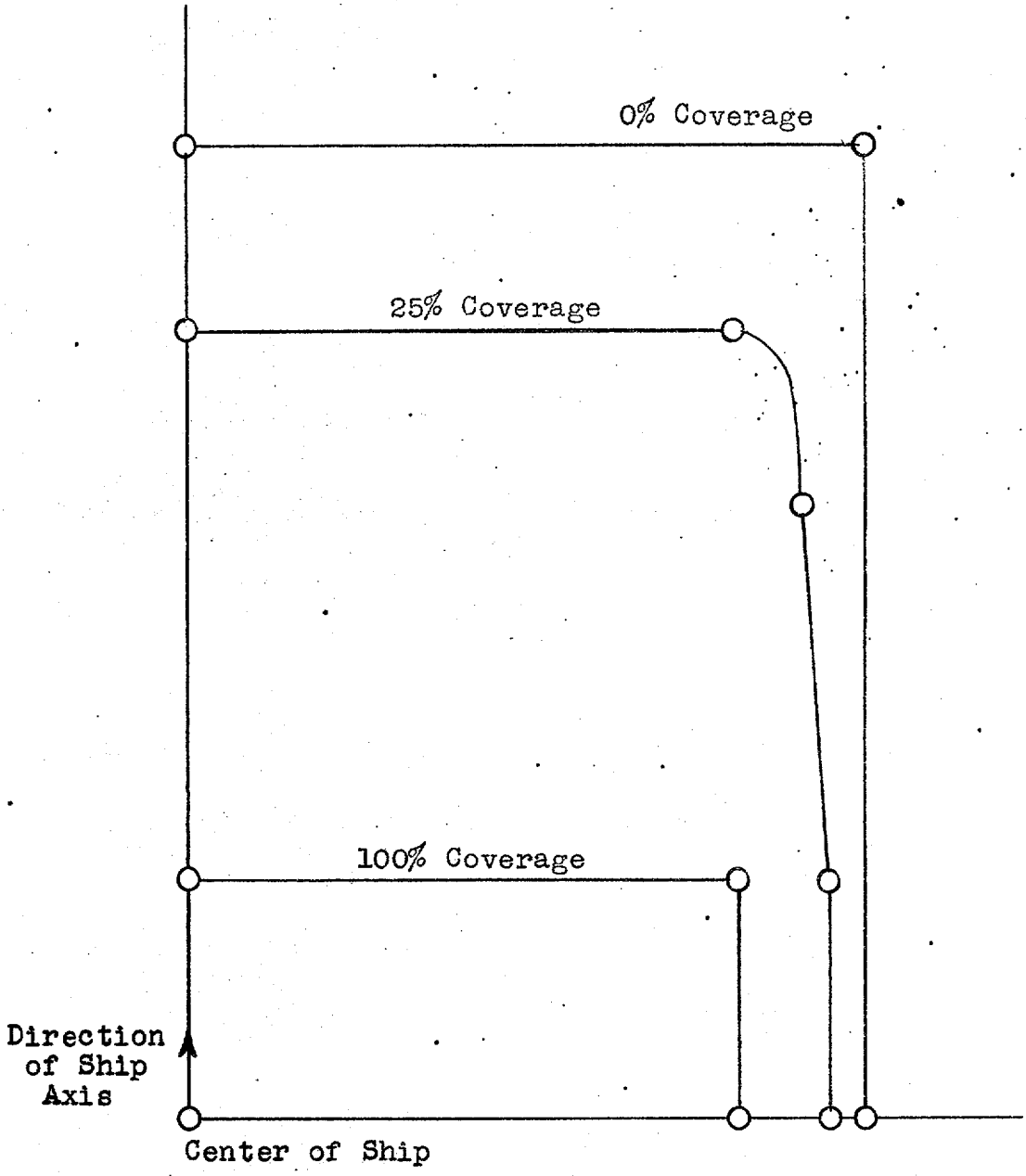


FIG. 2C LOCI OF CONSTANT TARGET COVERAGE BY BOMB PATTERN

(see pp. 27,28)

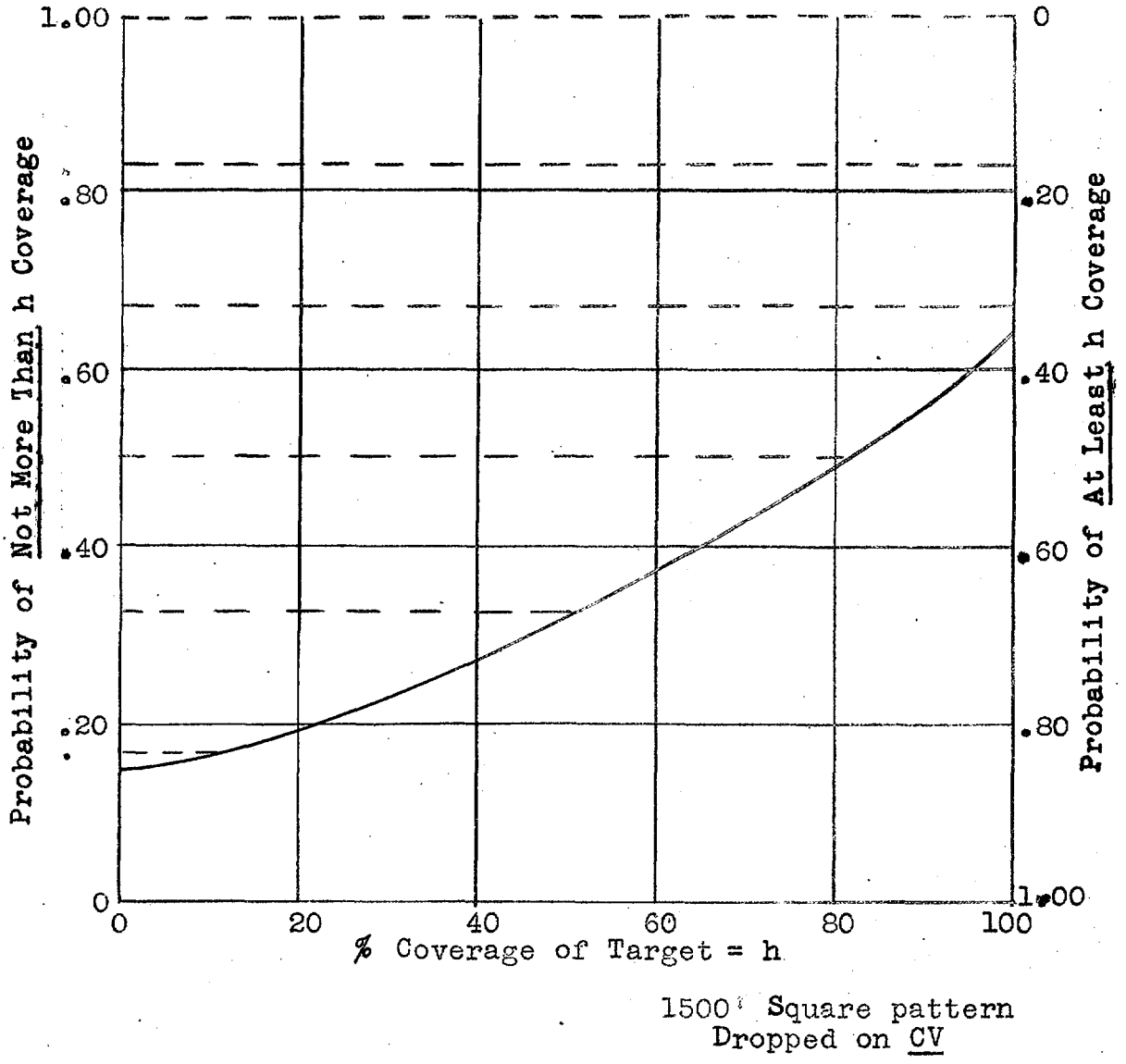


FIG. 3C PROBABILITY OF h % TARGET COVERAGE

Figure 3C may now be used to determine the 16.6, 33.3, 50.0,etc., percentiles:

PERCENTILES

	16.6	33.3	50.0	66.6	83.3	100.0
% Coverage	10	52	82	100	100	100

These figures indicate that one sixth of the time, the coverage will be between 0 and 10%; one sixth, between 10 and 52%; etc. The means of these limits are selected for the preparation of the table of percentage of hits. We may now write a a table of sextiles:

SEXTILES

	1	2	3	4	5	6
% Coverage	5	31	77	91	100	100

Multiplying each of the figures in the table by the ratio of the ship area to the pattern area gives the following table:

PERCENTAGE OF HITS IN EACH SEXTILE

	1	2	3	4	5	6
CEP 600'	0.3	1.9	4.1	5.5	6.0	6.0

Using a mil bombing error of 30 to 60 mils permits us to convert the CEP to equivalent bombing altitude. The sextile to be used in the individual case may be selected by the roll of a single die, resulting in the final form of the table:

PERCENTAGE OF HITS

Altitude	1	2	3	4	5	6
10,000 to 19,00 ft.	0.3	1.9	4.1	5.5	6.0	6.0

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